

Problem 5.2

The general solution is (Eq. 5.6):

$$y(t) = C_1 \cos(\omega t) + C_2 \sin(\omega t) + \frac{E}{B}t + C_3; \quad z(t) = C_2 \cos(\omega t) - C_1 \sin(\omega t) + C_4.$$

(a) $y(0) = z(0) = 0$; $\dot{y}(0) = E/B$; $\dot{z}(0) = 0$. Use these to determine C_1 , C_2 , C_3 , and C_4 .

$$y(0) = 0 \Rightarrow C_1 + C_3 = 0; \quad \dot{y}(0) = \omega C_2 + E/B = E/B \Rightarrow C_2 = 0; \quad z(0) = 0 \Rightarrow C_2 + C_4 = 0 \Rightarrow C_4 = 0;$$

$\dot{z}(0) = 0 \Rightarrow C_1 = 0$, and hence also $C_3 = 0$. So $y(t) = Et/B$; $z(t) = 0$. Does this make sense? The magnetic force is $q(\mathbf{v} \times \mathbf{B}) = -q(E/B)B\hat{z} = -q\mathbf{E}$, which exactly cancels the electric force; since there is no net force, the particle moves in a straight line at constant speed. ✓

(b) Assuming it starts from the origin, so $C_3 = -C_1$, $C_4 = -C_2$, we have $\dot{z}(0) = 0 \Rightarrow C_1 = 0 \Rightarrow C_3 = 0$;

$$\dot{y}(0) = \frac{E}{2B} \Rightarrow C_2\omega + \frac{E}{B} = \frac{E}{2B} \Rightarrow C_2 = -\frac{E}{2\omega B} = -C_4; \quad y(t) = -\frac{E}{2\omega B} \sin(\omega t) + \frac{E}{B}t;$$

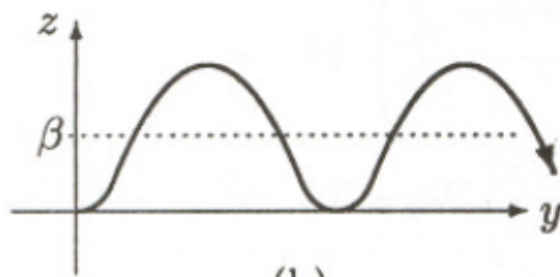
$$z(t) = -\frac{E}{2\omega B} \cos(\omega t) + \frac{E}{2\omega B}, \quad \text{or} \quad y(t) = \frac{E}{2\omega B} [2\omega t - \sin(\omega t)]; \quad z(t) = \frac{E}{2\omega B} [1 - \cos(\omega t)]. \quad \text{Let } \beta \equiv E/2\omega B.$$

Then $y(t) = \beta [2\omega t - \sin(\omega t)]$; $z(t) = \beta [1 - \cos(\omega t)]$; $(y - 2\beta\omega t) = -\beta \sin(\omega t)$, $(z - \beta) = -\beta \cos(\omega t) \Rightarrow (y - 2\beta\omega t)^2 + (z - \beta)^2 = \beta^2$. This is a circle of radius β whose center moves to the right at constant speed: $y_0 = 2\beta\omega t$; $z_0 = \beta$.

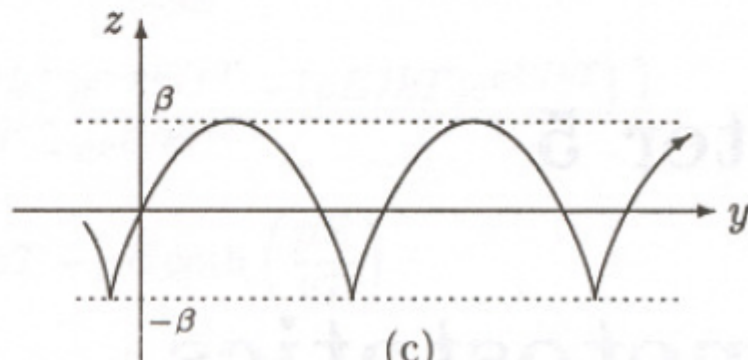
$$(c) \dot{z}(0) = \dot{y}(0) = \frac{E}{B} \Rightarrow -C_1\omega = \frac{E}{B} \Rightarrow C_1 = -C_3 = -\frac{E}{\omega B}; \quad C_2\omega + \frac{E}{B} = \frac{E}{B} \Rightarrow C_2 = C_4 = 0.$$

$$y(t) = -\frac{E}{\omega B} \cos(\omega t) + \frac{E}{B}t + \frac{E}{\omega B}; \quad z(t) = \frac{E}{\omega B} \sin(\omega t). \quad \boxed{y(t) = \frac{E}{\omega B} [1 + \omega t - \cos(\omega t)]; \quad z(t) = \frac{E}{\omega B} \sin(\omega t).}$$

Let $\beta \equiv E/\omega B$; then $[y - \beta(1 + \omega t)] = -\beta \cos(\omega t)$, $z = \beta \sin(\omega t)$; $[y - \beta(1 + \omega t)]^2 + z^2 = \beta^2$. This is a circle of radius β whose center is at $y_0 = \beta(1 + \omega t)$, $z_0 = 0$.



(b)



(c)

Problem 5.5

(a) $K = \frac{I}{2\pi a}$, because the length-perpendicular-to-flow is the circumference.

(b) $J = \frac{\alpha}{s} \Rightarrow I = \int J da = \alpha \int \frac{1}{s} s ds d\phi = 2\pi\alpha \int ds = 2\pi\alpha a \Rightarrow \alpha = \frac{I}{2\pi a}; J = \frac{I}{2\pi as}$.

Problem 5.13

$$(a) \oint \mathbf{B} \cdot d\mathbf{l} = B 2\pi s = \mu_0 I_{\text{enc}} \Rightarrow \mathbf{B} = \left\{ \begin{array}{ll} 0, & \text{for } s < a; \\ \frac{\mu_0 I}{2\pi s} \hat{\phi}, & \text{for } s > a. \end{array} \right\}$$

$$(b) J = ks; I = \int_0^a J da = \int_0^a ks(2\pi s) ds = \frac{2\pi ka^3}{3} \Rightarrow k = \frac{3I}{2\pi a^3}. \quad I_{\text{enc}} = \int_0^s J da = \int_0^s k\bar{s}(2\pi\bar{s}) d\bar{s} =$$

$$\frac{2\pi ks^3}{3} = I \frac{s^3}{a^3}, \text{ for } s < a; I_{\text{enc}} = I, \text{ for } s > a. \text{ So } \mathbf{B} = \left\{ \begin{array}{ll} \frac{\mu_0 I s^2}{2\pi a^3} \hat{\phi}, & \text{for } s < a; \\ \frac{\mu_0 I}{2\pi s} \hat{\phi}, & \text{for } s > a. \end{array} \right\}$$

Problem 5.23

$$A_\phi = k \Rightarrow \mathbf{B} = \nabla \times \mathbf{A} = \frac{1}{s} \frac{\partial}{\partial s} (sk) \hat{\mathbf{z}} = \frac{k}{s} \hat{\mathbf{z}}; \quad \mathbf{J} = \frac{1}{\mu_0} (\nabla \times \mathbf{B}) = \frac{1}{\mu_0} \left[-\frac{\partial}{\partial s} \left(\frac{k}{s} \right) \right] \hat{\phi} = \boxed{\frac{k}{\mu_0 s^2} \hat{\phi}}.$$
