Can you Hear and See a Quark-Gluon Plasma?

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Why all scientists interested in the Quark-Gluon Plasma owe gratitude to John Cramer ...
...because he chose to write *Einsteins Bridge* about the SSC, not about RHIC!
RHIC has not destroyed our world

... or has it?
The original QGP allegory

You can’t hear it

Don’t make bold claims about it

You can’t see it

CERN
Part 1

“Seeing” the QGP
Suppression Pattern: Baryons vs. Mesons

... the “proton puzzle” came as a complete surprise ...

- What makes baryons different from mesons?
Hadronization Mechanisms

Fragmentation

Baryon
Meson ≪ 1

Recombination

\[
\begin{align*}
\text{Baryon}\,\frac{p_M}{\text{Meson}} &\approx 1 \\
p_M &\approx 2p_Q \\
p_B &\approx 3p_Q
\end{align*}
\]
Sudden recombination picture

Transition time from QGP into vacuum (in rest frame of produced hadron) is:

$$\tau_f = d / \gamma = d \frac{m}{p_T}$$

Allows to ignore complex dynamics in hadronization region; corrections $O(m/p_T)^2$

**Not** gradual coalescence from dilute system !!!
Relativistic formulation

Relativistic formulation using hadron light-cone frame \((P = P_\parallel)\):

\[
d^3k = \frac{k^0}{k^+} dk^+ d^2k_\perp \quad \text{with} \quad k^+ = \frac{1}{\sqrt{2}} \left( k^0 + k_\parallel \right) \quad \text{and} \quad k^+ = xP^+
\]

\[
E \frac{dN_M}{d^3P} = \int d\Sigma \frac{P \cdot u}{(2\pi)^3} \sum_{\alpha,\beta} \int dx w_\alpha(R, xP^+)\bar{w}_\beta(R, (1-x)P^+) |\bar{\phi}_M(x)|^2
\]

\[
E \frac{dN_B}{d^3p} = \int d\Sigma \frac{P \cdot u}{(2\pi)^3} \sum_{\alpha,\beta,\gamma} \int dx dx' w_\alpha(R, xP^+)w_\beta(R, x'P^+)w_\gamma(R, (1-x-x')P^+) |\bar{\phi}_B(x, x')|^2
\]

For a thermal distribution, \(w(r, p) \sim \exp(-p \cdot v / T)\)

the hadron wavefunctions can be integrated out, eliminating the model dependence of predictions.

This is true even if higher Fock space states are included!
Recombination is favored ... 

... for a thermal source

Fragmentation always wins for a power law tail

Baryons compete with mesons
Model fit to RHIC hadron spectrum


\[ T_{\text{eff}} = 350 \text{ MeV} \]

Blue-shifted temperature

\( p\text{QCD spectrum shifted by 2.2 GeV} \)
Confronting RHIC data

- R+F model describes different $R_{AA}$ behavior of protons and pions
- Jet-quenching becomes universal in the fragmentation region
Collision Geometry: Elliptic Flow

Elliptic flow ($v_2$):

- Gradients of almond-shape surface will lead to preferential expansion in the reaction plane
- Anisotropy of emission is quantified by 2$^{\text{nd}}$ Fourier coefficient of angular distribution: $v_2$

- Bulk evolution described by relativistic fluid dynamics,
- assumes that the medium is in local thermal equilibrium,
- but no details of how equilibrium was reached.
- **Input:** $\varepsilon(x, \tau_i), P(\varepsilon), (\eta, \text{etc.})$. 
Quark Number Scaling of Elliptic Flow

In the recombination regime, meson and baryon $v_2$ can be obtained from the parton $v_2$ (using $x_i = 1/n$):

\[ v_2^M (p_t) = \frac{2v_2^p \left( \frac{p_t}{2} \right)}{1 + 2 \left( v_2^p \left( \frac{p_t}{2} \right) \right)^2} \]

and

\[ v_2^B (p_t) = \frac{3v_2^p \left( \frac{p_t}{3} \right) + 3 \left( v_2^p \left( \frac{p_t}{3} \right) \right)^3}{1 + 6 \left( v_2^p \left( \frac{p_t}{3} \right) \right)^2} \]

Neglecting quadratic and cubic terms, a simple scaling law holds:

\[ v_2^M (p_t) = 2v_2^p \left( \frac{p_t}{2} \right) \]

and

\[ v_2^B (p_t) = 3v_2^p \left( \frac{p_t}{3} \right) \]
Hadron $v_2$ reflects quark flow!
Higher Fock states don’t …

... spoil the analysis, they just modify the quark-hadron $v_2$ mapping

$$|M\rangle = C_1 |q\bar{q}\rangle + C_2 |q\bar{q}g\rangle$$

$$|B\rangle = C_1 |qqq\rangle + C_2 |qqqq\rangle$$

$$\phi_1^{(M)}(x_a, x_b) \sim x_a x_b$$

$$\phi_2^{(M)}(x_a, x_b, x_g) \sim x_a x_b x_g^2$$

$$\phi_1^{(B)}(x_a, x_b, x_c) \sim x_a x_b x_c$$

$$\phi_2^{(B)}(x_a, x_b, x_c, x_g) \sim x_a x_b x_c x_g^2$$

\[ C_2 = 0.3 \]
Hadron production at the LHC


equation

\[ r = 0.75 \]

\[ r = 0.85 \]

\[ r = 0.65 \]

includes parton energy loss

**\( \beta_r = 0.85 \)**

**\( \beta_r = 0.65 \)**

**\( \beta_r = 0.75 \)**

- \( \frac{1}{2\pi p_T} dN/dp_T \)
Part 2

Imaging the Fireball
HBT density interferometry

Two-particle wave function needs to account for the interactions among the two particles and between particles and the emitting medium, encoded in their optical potential.

\[ A_{12} = \frac{1}{\sqrt{2}} \left[ e^{i p_1 \cdot (r_1 - x)} e^{i p_2 \cdot (r_2 - y)} + e^{i p_1 \cdot (r_1 - y)} e^{i p_2 \cdot (r_2 - x)} \right] \]

so that

\[ \mathcal{P}_{12} = \int d^4x \, d^4y \, |A_{12}|^2 \rho(x) \rho(y) = 1 + |\tilde{\rho}(q)|^2 \equiv C_2(q) \]

Two-particle wave function needs to account for the interactions among the two particles and between particles and the emitting medium, encoded in their optical potential.

(JG Cramer and GA Miller)
Formalism

Two-particle emission function:

\[ S(x; p_1, p_2) = \int \frac{d^4 y}{2(2\pi)^3} \left< J^* (x + \frac{1}{2} y) J(x - \frac{1}{2} y) \right> \psi_{p_1}^{(-)}(x + \frac{1}{2} y) \psi_{p_2}^{(-)}(x - \frac{1}{2} y) \]

\[ J = \text{pion source} \]

Exact outgoing scattering solution:

\[ \left( \frac{\partial^2}{\partial t^2} - \nabla^2 + U_{\text{opt}} + m^2 \right) \psi_p^{(-)}(x) = 0 \]

Two-particle correlation function:

\[ C(p_1, p_2) = 1 + \frac{\left| \int d^4 x S(x; p_1, p_2) \right|^2}{\int d^4 x S(x; p_1) \int d^4 x S(x; p_2)} \]
Pion source fits

Au+Au at 100 GeV/u

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<th>$\Delta \tau$ (fm/c)</th>
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<td>±0.032</td>
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“Polishing” the lens

Full optical potential $U$

Re($U$) = 0

Eikonal ($U = 0$)

$K_T = 100$ MeV/c
$K_T = 250$ MeV/c
$K_T = 600$ MeV/c
Part 3

“Hearing” the QGP
Jet-medium interactions

- How does a fast parton interact with the quark-gluon plasma?

- What happens to the energy and momentum lost by a fast parton on its passage through the hot medium?

- How does the energy and momentum perturbation of the medium propagate?

Thanks to: E. Wenger (PHOBOS)

What happens here?!?
Where does the “lost” energy go?

Lost energy of away-side jet is redistributed to angles away from $180^\circ$ and low transverse momenta $p_T < 2 \text{ GeV/c}$ (Mach cone?).
Away side shape modification

Technique: Measure 2- and 3- particle correlations on the away-side triggered by “high” $p_T$ hadron in central collisions.

Cone-shaped emission shows up in 3-particle correlations as signal on both sides of the backward direction.
Parton-medium coupling

\[
\left[ \frac{p^\mu}{E} \frac{\partial}{\partial x^\mu} - \nabla_p \cdot D(x,p) \cdot \nabla_p \right] f_0(x,p) = C[f_0]
\]

with

\[
D_{ij}(x,p) = \int_{-\infty}^{t} dt' F_i(\bar{x},t) F_j(\bar{x} + \bar{v}(t'-t),t') .
\]

\[
\frac{\partial}{\partial x^\mu} T^{\mu\nu} = J^\nu
\]

with

\[
\begin{cases} 
T^{\mu\nu} = (\epsilon + p) u^\mu u^\nu - p g^{\mu\nu} + T^{\mu\nu}_{\text{diss}} \\
J^\nu = \int d\mathbf{p} \ p^\nu \nabla_p \cdot D(x,p) \cdot \nabla_p \ f(x,p)
\end{cases}
\]

Space-time distribution of collisional energy loss

Color field of moving parton interacts with the quanta of the medium
Energy density

$J^0(\rho,z)$ unscreened

$J^0(\rho,z)$ screened

(\text{GeV})^4

(z - ut) \rightarrow q

Bryon Neufeld

$u = 0.99$
Linearized hydro

Linearize hydro eqs. for a weak source: $T^{00} \rightarrow \varepsilon_0 + \delta \varepsilon$, $T^{0i} \rightarrow g^i$.

\[
\frac{\partial}{\partial t} \delta \varepsilon + \nabla \cdot \tilde{g} = J^0 \quad \frac{\partial}{\partial t} \tilde{g} + c_s^2 \nabla \delta \varepsilon + \frac{\eta}{\varepsilon_0 + p_0} \frac{4}{3} \nabla (\nabla \cdot \tilde{g}) = \tilde{J}
\]

Solve in Fourier space for longitudinal sound:

\[
\delta \varepsilon = i \frac{\left(\omega + i \Gamma_s k^2\right) J^0 + k J_L}{\omega^2 - c_s^2 k^2 + i \Gamma_s \omega k^2} \quad g_L = i \frac{c_s^2 k J^0 + \omega J_L}{\omega^2 - c_s^2 k^2 + i \Gamma_s \omega k^2}
\]

… and dissipative transverse perturbation:

\[
g_T = i \frac{J_T}{\omega + \frac{3}{4} i \Gamma_s k^2}
\]

Use: $u = 0.99955 c$, $c_s^2 = \frac{1}{3}$, $\Gamma_s = \frac{1}{3\pi T}$ for $T = 350$ MeV.
Contour plots

Mach cone

\[ \gamma = 30 \]

\[ \gamma = 0.99955 \, c \]
pQCD vs. $N=4$ SYM

$u = 0.99955 \, c$

Neufeld et al.
arXiv:0802.2254

$u = 0.75 \, c$

Chesler & Yaffe
arXiv:0712.0050

Friday, September 11, 2009
The ultimate “crescendo”

Radiative energy loss > collisional energy loss, but only collisions deposit energy into the plasma. However, radiated gluons contribute to the sound source:

The “soloist” becomes a chamber “orchestra”!

Energy deposit by quark is constant

Energy deposit by gluons grows with $L$

Energy deposit by gluons grows with $L$

$\frac{\partial}{\partial x} f(\omega, x) - \frac{\partial}{\partial \omega} [\varepsilon(\omega)f(\omega, x)] = \frac{dI(\omega, x)}{d\omega dx}$

The “crescendo” could explain why experiments show sound velocity $c_s = 0.3$ corresponding to $T_c$.

R.B. Neufeld & BM, PRL 103, 042301

Friday, September 11, 2009
Back-to-back partons

Linearized hydro simulation of radiation-enhanced Mach cone (R.B. Neufeld)
The QGP can be “seen” through the formation of hadrons via recombination of collectively flowing quarks.

“Slow” hadron quantum correlations reveal an image of the emitting source, which is sensitive to the hadron interactions with the medium.

Energetic partons (jet progenitors) produce a sonic Mach cone in the QGP, which grows with time and peaks at $T_c$.

Thus - can we hear and see the quark-gluon plasma?

Yes, we can!