Physics 517/617 Homework 4 (Due August 9th)

The following problem is designed to familiarize you with the concept of amplitude modulation (Horowitz and Hill 892-898). This concept is obviously crucial to the understanding of the AM radio you are about to build. The general expression for an Amplitude Modulated voltage is:

$$V(t) = \left[1 + a\cos\left(\omega_m t\right)\right]\cos\left(\omega_c t\right)$$

In this expression ω_c is the carrier frequency, ω_m is the modulating frequency and a is the amount of modulation, the modulation depth ($0 < a < 1$). For the AM radio example the carrier frequency, ω_c , is high frequency (hundreds of kHz) while the modulating, ω_m, frequency is low frequency: audio frequencies are 20Hz to 20 kHz, for the radio you make, frequencies out to about 3 kHz will be detected.

1) Make a sketch of V if one assumes ω_m = 1 KHz, ω_c = 10 Khz, and a=1.

2) Show that V can be written in the following form which contains three frequencies. Relate ω_1, ω_2, and ω_3 to ω_m and ω_c :

$$V(t) = \cos\left(\omega_1 t\right) + \frac{1}{2}\cos\left(\omega_2 t\right) + \frac{1}{2}\cos\left(\omega_3 t\right)$$

3) Show that for small voltages (V) the Ebers-Moll (or Diode) equation for current (I) has the following form:

$$I(t) = \alpha V(t) + \beta\left[V(t)\right]^2$$

where α and β are constants.

4) Assuming that the current is given by the expression in part 3. and the voltage is given by the expression in part 2. Show that the resulting current has a term that depends linearly on $\cos\omega_m t$ and a term that depends linearly on $\cos\omega_c t$ (it also has many other terms).

5) Use the results of part 4. to describe how a high frequency AM signal gets demodulated (turned into audio frequencies) in the radio you will be building in lab. Which frequencies are amplified and which are filtered out (by the frequency response of the non-inverting op amp amplifier)?