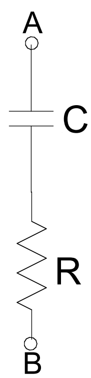
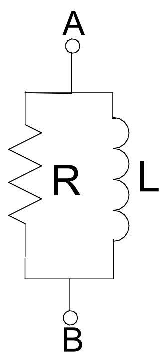
Physics 517/617 Homework 2

Problems for AC circuits

1) Calculate the impedance Z_{AB} in the form $a+jb$ and $|Z|e^{j\phi}$ for the following circuits:

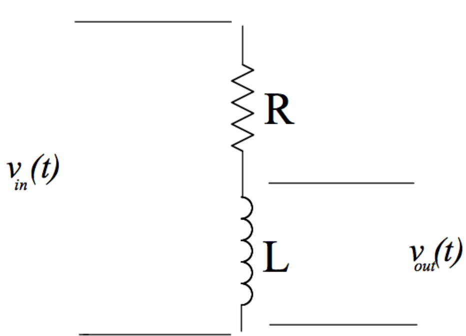
a) 

b) 

a)
$$Z = R + j\left(-\frac{1}{\omega C}\right)$$
$$Z = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2} \bullet e^{j\bullet\tan^{-1}\left[-\left(\frac{1}{\omega RC}\right)\right]}$$

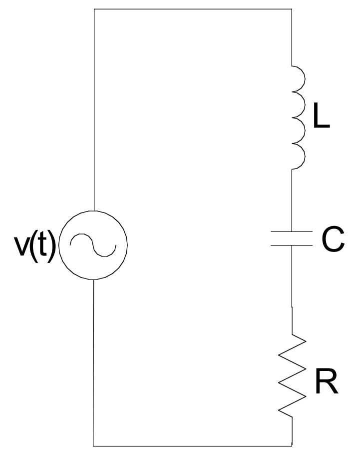
b) $$Z = \frac{\omega^2 L^2 R}{\omega^2 L^2 + R^2} + j\frac{\omega L R^2}{\omega^2 L^2 + R^2} = \frac{\omega L R}{\sqrt{\omega^2 L^2 + R^2}} e^{j\tan^{-1}\left[\frac{R}{\omega L}\right]}$$

2) Design a high-pass RL filter with a 3 dB point of 100 kHz. Use a 1 $k\Omega$ resistance. Explain in words why the high-pass filter attenuates the low frequencies.



Need $\frac{R}{L} = 2\pi\bullet 100000$, which for a 1 $k\Omega$ resistance requires L=1.6 mH. The circuit can be thought of as a frequency dependent voltage divider. As $\omega \to 0$, the impedance of the inductance goes to zero and in turn the fraction of the input across the inductor goes to 0.

3) For the circuit shown at the right, $L = 50$ mH, $C = 200$ pF, and $R = 100\ \Omega$, and $v(t) = V_0 \cos\omega t$ with $V_0 = 10$ V and ω is adjustable. Find (a) the resonant frequency of the circuit, f_0, (b) the Q of the circuit, (c) the amplitude of the voltage across the resistor, the inductor, and the capacitor, V_R, V_L, and V_C, when the circuit is driven at its resonant frequency.



a) $f = \dfrac{1}{2\pi\sqrt{LC}} = 50$ kHz

b) $Q = \dfrac{\omega L}{R} = 157$

On resonance, the impedances of the inductor and the capacitor are both large in magnitude, but equal and opposite in sign --- they sum to give zero impedance, Thus the voltage across the reistor is just the input voltage, 10 V in this case and the voltages across the capacitor and inductor are both Q times this = 1570 V, but they have opposite phases.

4) For each of the following circuits identify the corresponding magnitude Bode plot. For most cases the Bode plot can be identified by considering the limits $\omega \to 0$ and $\omega \to \infty$.

a) $\Leftrightarrow$ 5)

b) $\Leftrightarrow$ 6)

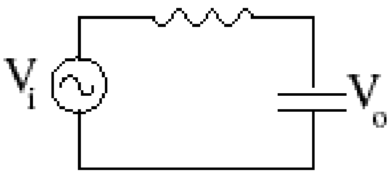
c) $\Leftrightarrow$ 4)

d) $\Leftrightarrow$ 2)

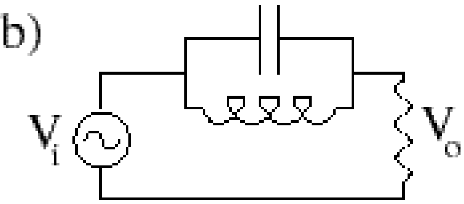
e) $\Leftrightarrow$ 1)

f) $\Leftrightarrow$ 3)

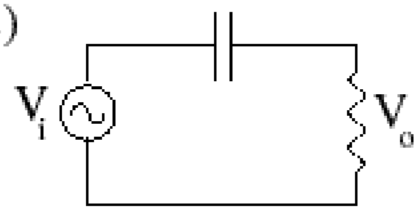
a)



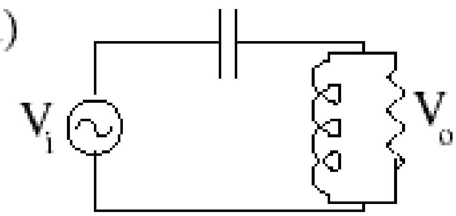
b)



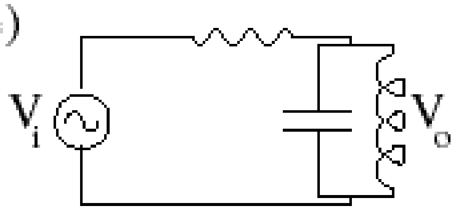
c)



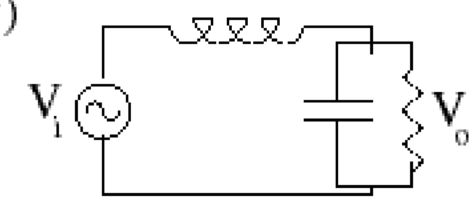
d)



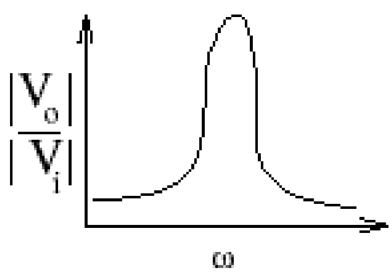
e)



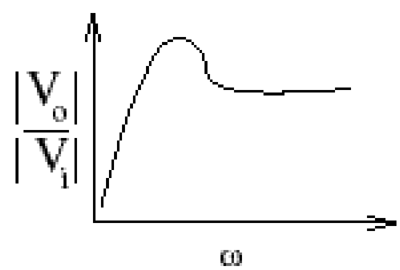
f)



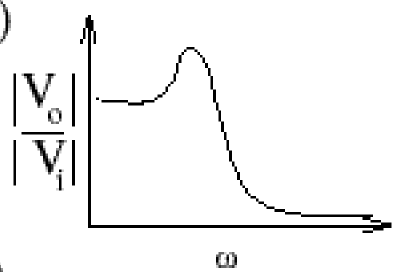
1)



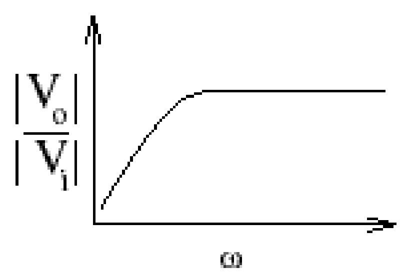
2)



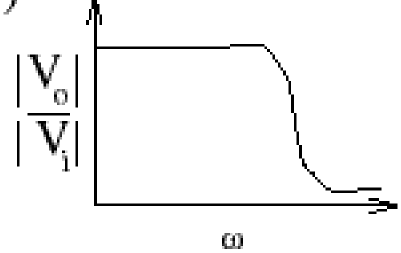
3)



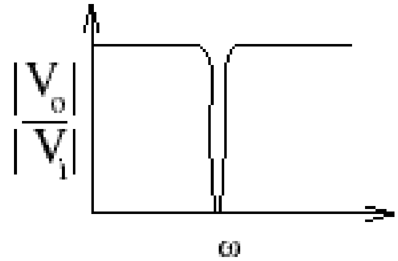
4)



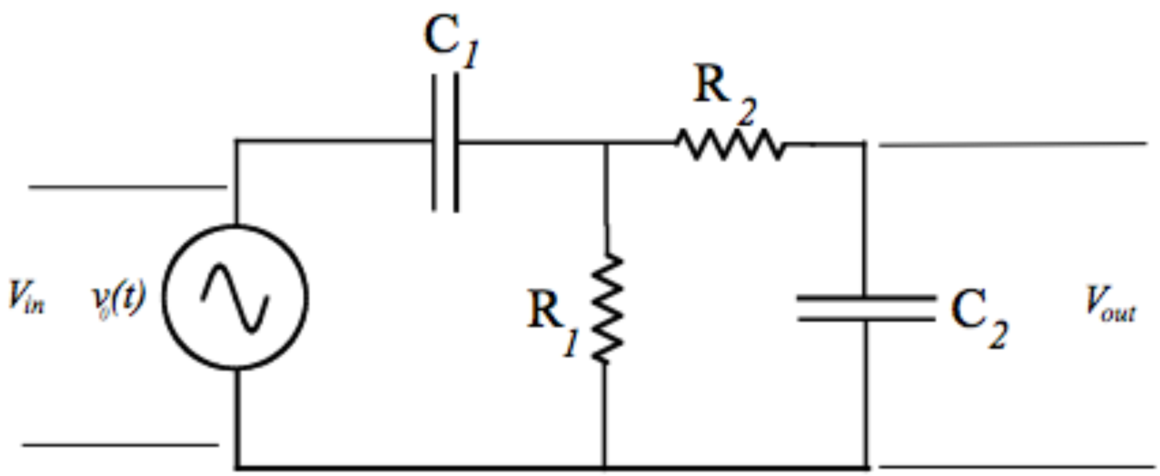
5)



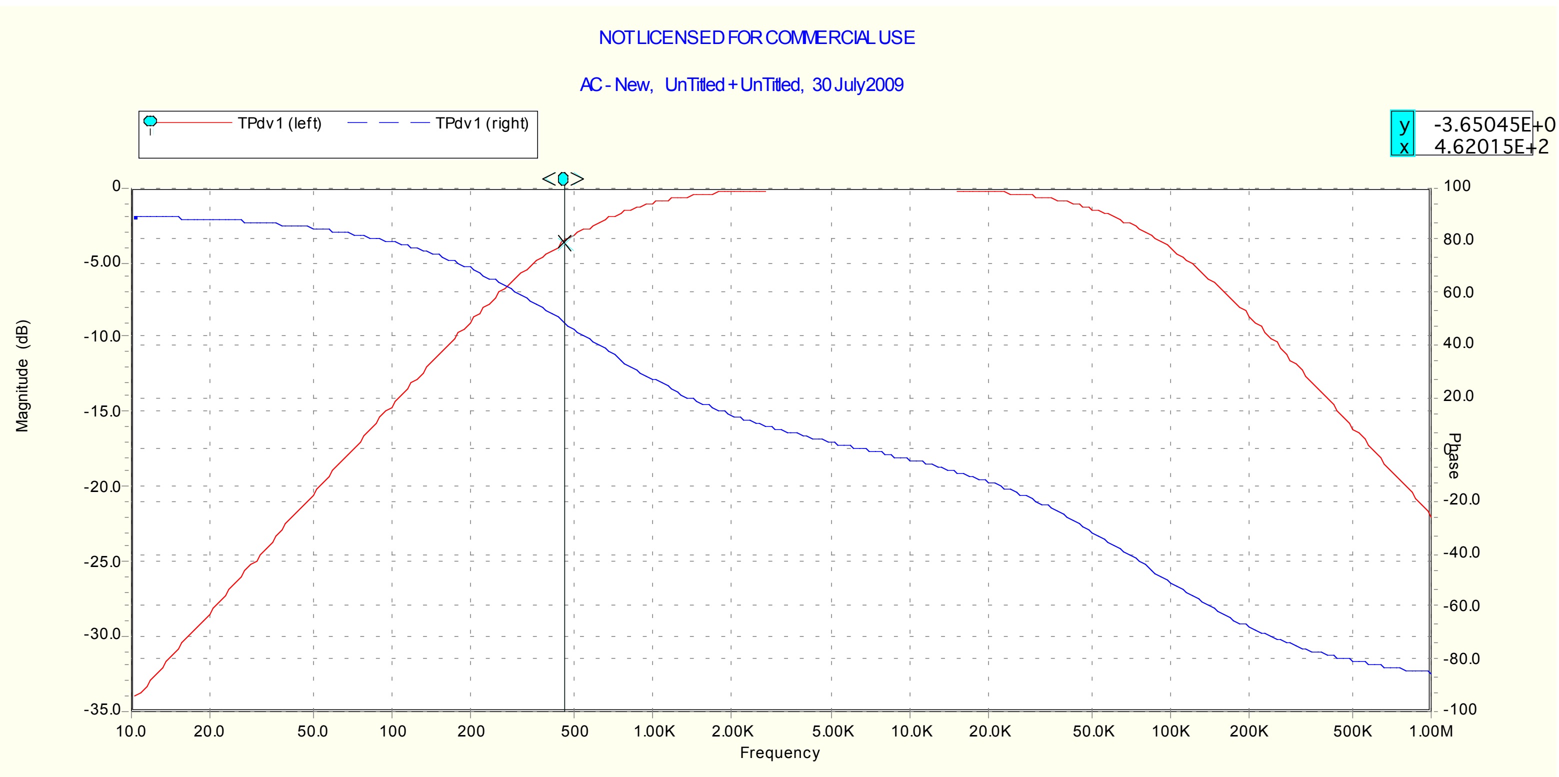
6)



5) Use SPICE to make Bode plots of the magnitude (gain) and phase of the transfer function for the filter shown below for frequencies from 10 Hz to 1 MHz. . Apply your results to the specific case of a 1.5 kHz, 2.0 V amplitude input to the filter , i.e., $v_0(t) = V_0 \cos(\omega t) = 2\cos(2\pi \cdot 1.5 \cdot 10^3 \cdot t)$. Find (a) the voltage at the output across C_2 and (b) the (numeric) value of the voltage at the output for time, $t = 0.001$ s.



$C_1 = 0.3\mu F \qquad R_1 = 1k\Omega \qquad C_2 = 100\,pF \qquad R_2 = 20k\Omega$



From the graphs for 1.5 kHz (note in problem, the original equation was mistakenly written as 150 Hz instead of 1500 Hz --- we won't take off for errors caused by that), the Magnitude is -2dB and the phase is $17°$. Therefore, the complex representation is given by:

$$\underset{\sim}{v}_{C2}(t) = \left|\underset{\sim}{V}_{C2}\right| e^{j\omega t + \varphi_{C2}}$$

Where $20\log\left|\frac{\underset{\sim}{V}_{C2}}{V_{in}}\right| = -2 \quad \Rightarrow \quad \left|\underset{\sim}{V}_{C2}\right| = 2 \cdot 10^{-0.1} = 1.6$. And $\varphi_{C2} = 17° = 0.3\,rad$. Taking the real part of this:

$$v(t = 0.001) = 1.6 \cdot \cos\left(2\pi \cdot 1500 \cdot 0.001 + 0.3\right) = -1.9\,V$$