# Summary Lecture 16

## Rotational and Linear Motion

### Displacement

<table>
<thead>
<tr>
<th>Linear:</th>
<th>$\Delta x$, measured in [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rotational:</td>
<td>$\Delta \theta$, difference in angle measured in [rad]</td>
</tr>
</tbody>
</table>

360 degrees = $2\pi$ rad

**Conversion:**

$\theta [\text{rad}] = \theta [\text{degrees}] \times \frac{2\pi}{360}$

**Arc-length $\ell$:**

$\ell = \theta r$; $\theta$ in [rad]

### Velocity

<table>
<thead>
<tr>
<th>Linear:</th>
<th>$v = \Delta x/\Delta t$, measured in [m/s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rotational:</td>
<td>$\omega = \Delta \theta/\Delta t$, measured in [rad/s]</td>
</tr>
</tbody>
</table>

**Period**

$T = \text{time for one revolution}$

**Frequency**

$f = \text{# of revs per second}$

$f = \frac{1}{T}$

$\omega = \frac{2\pi}{T} = 2\pi f$

### Acceleration

<table>
<thead>
<tr>
<th>Linear:</th>
<th>$a = \Delta v/\Delta t$, measured in [m/s²]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rotational:</td>
<td>$\alpha = \Delta \omega/\Delta t$; measured in [rad/s²]</td>
</tr>
</tbody>
</table>
Uniformly Accelerated Motion: (constant acceleration)

**Linear**
- \( v = v_0 + at \)
- \( x = x_0 + v_0 t + \frac{1}{2}at^2 \)
- \( v^2 = v_0^2 + 2a\Delta x \)
- \( \bar{v} = (v + v_0)/2 \)

**Rotational**
- \( \omega = \omega_0 + \alpha t \)
- \( \theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2 \)
- \( \omega^2 = \omega_0^2 + 2\alpha \Delta \theta \)
- \( \bar{\omega} = (\omega + \omega_0)/2 \)

+ centripetal acceleration
- \( a_c = \frac{v^2}{r} = \omega^2 r \)
The Object vs. Individual Points

Frequency, period, angular velocity and angular acceleration are properties of the body as a whole, i.e. every point moves with the same angular velocity. \((T, \, f, \, \omega, \, \alpha)\)

Each individual point also has a linear velocity, a linear acceleration and as always in circular motion a centripetal acceleration:

\[
\begin{align*}
v_T &= \frac{\Delta x}{\Delta t} = \frac{2\pi r}{T} = \omega r \\
a_T &= \frac{\Delta v}{\Delta t} = \frac{\Delta \omega}{\Delta t} \, r = \alpha r \\
a_C &= \frac{v_T^2}{r} = \omega^2 \, r
\end{align*}
\]