Physics 263: Problem Set #10

These problems are due at the end of the day on Friday, May 5.

0. Unassigned homework: go over old homework problems and solutions. Key math techniques you'll want: expansion in Taylor series, Fourier series, various integration techniques (parts, parameters), complex numbers and functions thereof, matrices (inverses, determinants, eigenvectors). Key physics contexts: basic relativity kinematics, $E^2 = p^2 + m^2$ collision problems, oscillators (whether damped, driven or coupled).

1. Shankar, BTM problem 6.4.5 pg. 138

2. Shankar, BTM problem 9.7.10 pg. 288.

3. Shankar, BTM problem 9.7.14 pg. 292.

4. Morin 12.30 (Maximum mass) p. 617. "how should it be divided" means: given m and the total E where $E = E_m + E_\gamma$, what choice of E_m and E_γ maximizes the mass of the resulting system.

5. (a) Expand the following function about $z = 0$ through order z^3 (keeping *only* up to z^3)
$$\frac{\cosh(z)\sin(z)}{\sqrt{c^2 - z^2}}$$
   (b) Given $f(z) = u(x, y) + iv(x, y)$, where $u(x, y) = x^2 - y^2 + 1 + 2x$, find a $v(x, y)$ which makes $f(z)$ analytic. What is this $f(z)$ (i.e. written explicitly as a function of z)? Is this $f(z)$ unique? Explain.

6. (a) Express the complex number $\sqrt{-i}$ in both polar and in $x + iy$ format. Same for $\cosh[1 + i\pi]$.
   (b) Solve the equations $x - 2y = 3$ and $3x + 2y = 1$ for x and y by matrix inversion.

7. Consider the function
$$f(x, y) = \frac{1}{3}x^3 + y^2 - xy.$$
   (a) Locate any critical points in the x-y plane.

(b) Compute the Hessian matrix (the matrix of 2nd derivatives), and evaluate it at each critical point.

(c) Find the eigenvalues of the Hessian, and decide whether each critical point is a maximum, a minimum or a saddle point.

(d) Finish this out by finding the eigendirections.

8. Consider the space of all functions on the interval $x \in (-1, 1)$. We choose as our basis vectors the polynomials $P_n(x)$ (for $n = 0$ and up) defined by the following rules: (a) $P_n(x)$ is an nth degree polynomial in x, (b) the P_n form an orthonormal basis (i.e. they are orthogonal to each other, and are properly normalized.)

   (a) Find $P_n(x)$ for n from 0 to 2.

   (b) Find the a_0, a_1 and a_2 which appear in the expansion $\exp(x) = \sum_{n=0}^{\infty} a_n P_n(x)$.

9. (BONUS) Animate something. That is, pick some physics problem, find the equations of motion, and then solve them, possibly using NDSolve[]. You can use some problem we have previously solved, or try something new. Make a Mathematica notebook, and print it out to hand in with the paper homework. Also please mail a copy to kilcup@physics.ohio-state.edu and I'll post a few of the animations on the website. Here are few possibilities from chapter 6:

   (a) 6.14 (Pendulum with free support) and/or the same thing sliding down (6.15)

   (b) 6.19 (Double pendulum) Why not finish year with a 4-star problem? (At least it is not an "exercise.")

   (c) 6.24 (Brachistochrone) Race beads on different curves, showing that the brachistochrone always wins.

   (d) 6.41 (Spring and a wheel)

   (e) 6.42 (Spring on a spoke) Illustrate the motion for both the magic value of k and a non-magic one.

   (f) 6.45 (Mass sliding on a rim) Use NDSolve to illustrate the general motion, but also show the eigenmodes for small oscillations (when $\sin\theta \approx \theta$ and the problem can be done "by hand.")

   (g) Or perhaps add up 2D Fourier modes to visualize the motion of a circular drumhead.