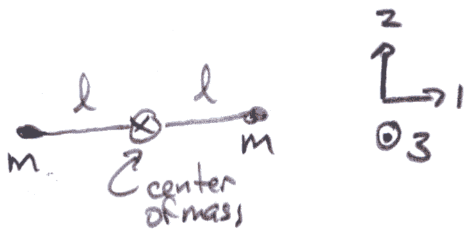
Physics 262 Worksheet #2

Yes hand this in w/ the homework.

Consider two point masses at the ends of a massless rigid rod of length $2l$: We introduce a coordinate basis $\hat{e}_1, \hat{e}_2, \hat{e}_3$.



(a) Suppose the rod spins with $\vec{\omega} = \omega_2 \hat{e}_2$

Then $\vec{L} =$

(b) Suppose instead $\vec{\omega} = \omega_1 \hat{e}_1$. Then $\vec{L} =$

(c) And if $\vec{\omega} = \omega_3 \hat{e}_3$ then $\vec{L} =$

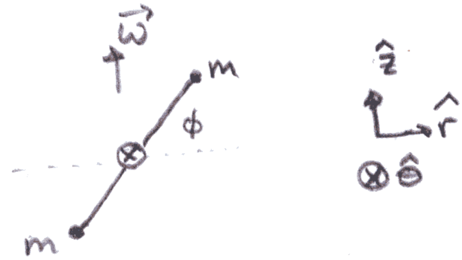
(d) Suppose for a moment that the "point" masses are discovered to be tiny spheres with radius b. How does that affect $\vec{L}$?

$\omega_1 \hat{e}_1 \Rightarrow \vec{L} =$

$\omega_2 \hat{e}_2 \Rightarrow \vec{L} =$

$\omega_3 \hat{e}_3 \Rightarrow \vec{L} =$

(e) Now set $b=0$ again, but let the rod tilt at an angle ϕ. We use a basis $\hat{r}, \hat{\theta}, \hat{z}$. Suppose $\vec{\omega} = \omega \hat{z}$.



First let us compute $\vec{L}$ the way we did before:

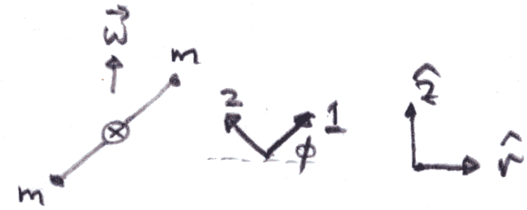
For one of the masses, $\vec{r}_1 =$

and $\vec{p}_1 =$ so $\vec{L}_1 = \vec{r}_1 \times \vec{p}_1 =$

Since $\vec{r}_2 = -\vec{r}_1$ and $\vec{p}_2 = -\vec{p}_1$ the other particle has the same $\vec{L}_2 = \vec{L}_1$,

so the total $\vec{L} = \vec{L}_1 + \vec{L}_2 =$

(f) Now reintroduce $\hat{e}_1$ and $\hat{e}_2$, still aligned with the natural axes of the rod:



From the geometry,

$$\hat{e}_1 = \boxed{\quad\quad}\,\hat{r} + \boxed{\quad\quad}\,\hat{z}$$

$$\hat{e}_2 = \boxed{\quad\quad}\,\hat{r} + \boxed{\quad\quad}\,\hat{z}$$

(Checks: make sure $\hat{e}_1 \cdot \hat{e}_1 = 1 = \hat{e}_2 \cdot \hat{e}_2$ and that $\hat{e}_1 \cdot \hat{e}_2 = 0$)

Likewise we can express $\hat{r}$ and $\hat{z}$ in terms of the $\hat{e}_1$ $\hat{e}_2$ basis:

$$\hat{r} = \boxed{\quad\quad}\,\hat{e}_1 + \boxed{\quad\quad}\,\hat{e}_2$$

$$\hat{z} = \boxed{\quad\quad}\,\hat{e}_1 + \boxed{\quad\quad}\,\hat{e}_2$$

(Again, is $\hat{r} \cdot \hat{z} = 0$ etc? And does $\hat{r} \cdot \hat{e}_1 = \hat{e}_1 \cdot \hat{r}$ etc when you compare both sets of relations?)

(g) So if $\vec{\omega} = \omega \hat{z} = \boxed{\quad\quad}\,\hat{e}_1 + \boxed{\quad\quad}\,\hat{e}_2$

Then $\vec{L} = I_1 \omega_1 \hat{e}_1 + I_2 \omega_2 \hat{e}_2 = \boxed{\quad\quad}\,\hat{e}_1 + \boxed{\quad\quad}\,\hat{e}_2$

Using the expressions above for $\hat{e}_1$ & $\hat{e}_2$ we can then express $\vec{L}$ as

$$\vec{L} = \boxed{\quad\quad}\,\hat{r} + \boxed{\quad\quad}\,\hat{z}$$

Hopefully this agrees with the first page.

(h) Lastly, <u>if</u> $\vec{\omega}$ is constant in time (generally it won't be) then there must be a torque on the rod:

$$\vec{\tau} = \frac{d}{dt}\vec{L} = \boxed{\quad\quad\quad\quad}$$

How does all this change when we replace the baton by a uniform skinny rod? (Answer: aside from a fraction or two, not much)