BTM 8.2.1 Given $M \equiv \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}$, the claim is that $M^{-1} = \frac{1}{\det M} \begin{bmatrix} M_{22} & -M_{12} \\ -M_{21} & M_{11} \end{bmatrix}$

Let us check:

$$M^{-1} \cdot M = \frac{1}{\det M} \begin{bmatrix} (M_{11}M_{22} + M_{12}(-M_{21})) & (M_{11}(-M_{12}) + M_{12}M_{11}) \\ (M_{21}M_{22} + M_{22}(-M_{21})) & (M_{21}(-M_{12}) + M_{22}M_{11}) \end{bmatrix}$$

$$= \frac{1}{\det M} \begin{bmatrix} \det M & 0 \\ 0 & \det M \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \checkmark$$

BTM 8.2.2 Applying this inverse formula to $R_\theta \equiv \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ which has $\det[R_\theta] = \cos\theta \cdot \cos\theta - (-\sin\theta)(\sin\theta) = 1$, we find

$$[R_\theta]^{-1} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} = R_{-\theta}$$

which is just what one expects — the inverse of a rotation by θ is a rotation by $-\theta$.

BTM 8.2.3 To solve the system

$$x_1 + 2x_2 = 9$$
$$3x_1 + 4x_2 = 23$$

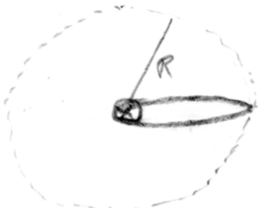
we write the system as $M \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 9 \\ 23 \end{bmatrix}$ where $M \equiv \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

Since $\det M = 4 - 6 = -2$, $M^{-1} = \frac{-1}{2} \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix}$

Then

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = M^{-1} \cdot M \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = M^{-1} \begin{bmatrix} 9 \\ 23 \end{bmatrix} = \begin{bmatrix} -18 + 23 \\ \frac{27}{2} - \frac{23}{2} \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$

Morin 7.12



How long would it take an object to fall into the sun starting from rest at the earth's distance from the sun?

Per suggestion, we think of falling radially in as the limiting case of the problem with some finite (but small) tangential starting velocity. For any nonzero ℓ we get an ellipse with a major axis approaching R. Kepler tells us that the period of such an ellipse satisfies $T^2 = kA^3$ where the constant k is such that it takes one year to orbit with the earth's major axis (which is $2R$), i.e. $T_0^2 = k(2R)^3$.

$$\Rightarrow T^2 = \frac{T_0^2}{(2R)^3} R^3 \Rightarrow T = \left(\tfrac{1}{2}\right)^{3/2} T_0$$

But the falling <u>in</u> portion is only $\frac{1}{2}$ of the period, so the time to incineration is $\left(\frac{1}{2}\right)^{5/2} = \frac{\sqrt{2}}{8}$ year $\sim$ 2 months