1. (*20 points*) Initially a small satellite is in circular orbit about Earth at a radius of r_0. Its thrusters fire briefly in either the negative radial (inward) or the positive tangential (along the velocity) direction, injecting an impulse putting the satellite into a new elliptical orbit. Mark each of the statements true or false. (Two points for a correct guess; two more points for the explanation behind the response.)

   T F If the blow is tangential there is an increase in the angular momentum.

   T F If the blow is radial there is no change in the energy.

   T F The eccentricity (ϵ) of the new orbit is greater than one.

   T F In the tangential case the minimum radius of the new orbit is equal to r_0.

   T F In the radial case the period of the new orbit is shorter than the original orbit.

2. (*10 points*) A thin uniform rod of mass M and length L is starts at rest above an anchored frictionless pivot point as drawn, and is then allowed to fall to the ground, thanks to gravity of strength g. With what speed does **the end of the rod** strike the ground?

3. (*25 points*) Consider two skaters, each of mass m, on frictionless ice. One moves with speed $3v_0$ to the right, while the other starts with speed v_0 to the left. Their paths are separated by distance b. At $t = 0$, when they are both at $x = 0$, they grasp a pole of length b and mass M. For $t > 0$ consider the system as a rigid body. We analyze the system using two choice of origin: A at particle 1 and B at the center of mass.

   (a) Which of these quantities do we expect to be conserved in the collision: K, $\vec{P}$, L_A and/or L_B?

   (b) Calling the final state variables v and ω as drawn, write down expressions for L_A and L_B in the final state.

   (c) Find these quantities v and ω.

   (d) What is the change in kinetic energy, if any?

4. (*20 points*) A satellite of mass m is moving in a **circular** orbit of radius $2R$ around the earth where R is the earth's radius. (The Earth's mass M, and Newton's constant G are also input parameters.) At an instant in time (that is, very quickly), the **direction** of motion of the satellite is changed through an angle α toward the earth, *without changing its speed and energy.* (The angle α is measured from the tangent to the original circular orbit, see figure.)

   (a) Express the energy and angular momentum of the satellite before and after the change in direction.

   (b) Sketch the old and new effective potentials on the same plot and draw a line to indicate the energy.

   (c) Find the angle(s) α such that the satellite just grazes the earth (i.e. such that $r = R$ is a turning point in the new V_{eff}.)

5. (*20 points*)

   Consider a uniform disk of radius R and mass m.

   (a) Demonstrate that the moment of inertia about its center is in fact $mR^2/2$ by breaking up the disk in to rings of thickness dr, each of which has $dI = dmr^2$, and doing the integral.

   (b) If the disk is made in to a pendulum by pivoting about a point A a distance x from the center, what is the *period* of small amplitude oscillations about $\phi = 0$?

(c) How should you choose x to minimize the period?

6. (*20 points*) Consider the ice-skaters again, i.e. two masses m connected by a massless rope of length b. If the two-particle system has CM velocity $v\hat{x}$ and spins with ω (counterclockwise) and if we choose coordinates so that at $t = 0$ mass 1 is at the origin and the other is at $b\hat{y}$,

   (a) Find the initial velocities $\vec{v}_1(0)$ and $\vec{v}_2(0)$.

   (b) Write good vector expressions for the positions $\vec{r}_1(t)$ and $\vec{r}_2(t)$ valid at general time t.

   (c) Considering particle 1 only, what is its angular momentum $\vec{L}_1(t)$?

   (d) What is $\frac{d}{dt}\vec{L}_1$?

   (e) If we call the tension in the rope T, find the torque $\vec{\tau}_1(t)$ on particle 1 (compute it explicitly as $\vec{r}(t) \times \vec{F}(t)$) and then determine T by comparing with the previous part.