Math Session #2

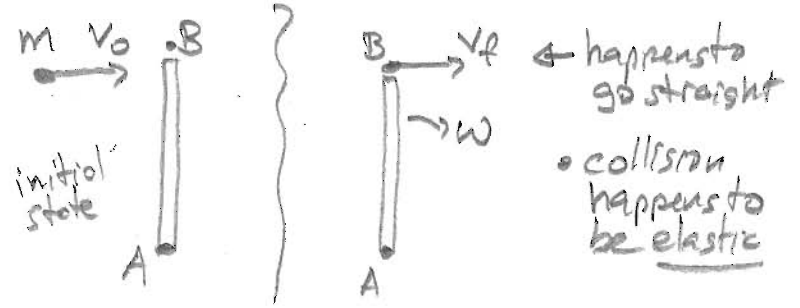
Here are some random matrices: $A \equiv \begin{bmatrix} 3 & -4 \\ -10 & -3 \end{bmatrix}$ $B \equiv \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

Compute $A+B = \begin{bmatrix} \; \end{bmatrix}$ and $A \cdot B = \begin{bmatrix} \; \end{bmatrix}$

fill in the dots

Have Mathematica do the same, starting with A = {{·, ·},{·, ·}} etc.
Also ask for A*B (which is different from A.B). Also ask for A.A

Recall the collision problem from last week: a mass m collides w/ a pivoted stick of mass M, length L.



In the collision process a large force F is applied to the stick at the pivot point, changing the momentum of the system by $\Delta p = \int F dt$
integral is over the short collision time

eqnP: $mv_0 + \Delta p = mv_f + MV$

By choosing origin A on top of the pivot we never need this equation; the two relations saying L_A and K are conserved suffice:

eqnK: $\frac{1}{2}mv_0^2 =$

eqnL_A: $-mv_0 L =$

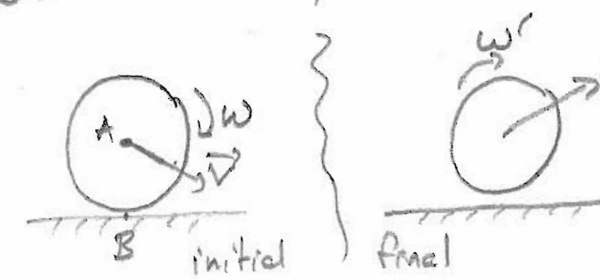
Have Mathematica Solve[] these to find v_f and ω.

If we choose origin B, the pivot force gives an angular impulse:

eqnL_B: $\underset{\text{initial } L_B}{0} + \Delta L = L\Delta p =$

Complete the equation and Solve for v_f, ω and Δp, noting of course that the CM of the stick moves with $V = \frac{L}{2}\omega$

Morin 8.20 considers a superball of mass m, radius R and moment of inertia I_0 $(=\frac{2}{5}mR^2)$. It spins at ω as it falls to the ground, and collides w/ the ground with velocity $\vec{v}=(v_x, v_y)$. Assuming that (1) the collision is elastic, (2) the outgoing v_y' is $-v_y$ and (3) no slipping during the collision (this is really part of "elastic", i.e. no K is lost) the goal is to find the final state (v_x', v_y') and spin rate ω'



One relation will be

$$\text{eqnK}: \quad \tfrac{1}{2} m v_x^2 + \tfrac{1}{2} I_0 \omega^2 = \tfrac{1}{2} m (v_x')^2 + \tfrac{1}{2} I_0 (\omega')^2$$

And noting the impulse $\Delta\vec{p} = (\Delta p_x, \Delta p_y)$ delivered to the system at point B,

$$\text{eqn}P_x: \quad m v_x + \Delta p_x = m v_x'$$

choosing origin A the angular momentum relation reads:

$$\text{eqn}L_A: \quad \pm I_0 \omega \pm R \Delta p_x =$$

you decide

Solve to find v_x', ω' and Δp_x

Choosing origin B, the force applied produces no torque, so we get

$$\text{eqn}L_B: \quad \pm m v_x R \pm I_0 \omega =$$

Again, complete and Solve the appropriate equations to find ω' & v_x'

Express the result in matrix form: $\begin{bmatrix} v_x' \\ R\omega' \end{bmatrix} = \begin{bmatrix} \quad \end{bmatrix} \begin{bmatrix} v_x \\ R\omega \end{bmatrix}$

call this M

Then answer Morin 8.21 by computing M^2.

Challenge: open the Math notebook for the day to define a drawing function, then Animate this motion. You will need to construct appropriate Piecewise[] continuous functions x[t_] etc.