Name: ____________________________

Physics 261: Simultaneous Equations Worksheet

The straightforward, guaranteed method to solve systems of linear equations is the successive elimination of variables.

Basic plan:

1. Pick a variable to eliminate and solve for it in one of the equations.
2. Replace that variable in *all* of the remaining equations.
3. With the remaining equations, repeat steps 1 and 2. Stop when only one equation remains.
4. Substitute in reverse order to determine all of the unknowns in terms of knowns.

Example:[some pulley problem, KK2.15]. Goal: Find T. The unknowns are T, $\ddot{x}_1$, $\ddot{x}_2$, and $\ddot{y}_3$. The equations are:

$$T - \mu m_1 g = m_1 \ddot{x}_1 \qquad (a) \qquad\qquad m_3 g - 2T = m_3 \ddot{y}_3 \qquad (c)$$

$$T - \mu m_2 g = m_2 \ddot{x}_2 \qquad (b) \qquad\qquad 2\ddot{y}_3 - \ddot{x}_1 - \ddot{x}_2 = 0 \qquad (d)$$

Since we're looking for T, we'll eliminate the other variables in turn, repeating steps 1 and 2 three times.

1. $(a) \Rightarrow \ddot{x}_1 = \dfrac{T - \mu m_1 g}{m_1}$

1. $(b) \Rightarrow \ddot{x}_2 = \dfrac{T - \mu m_2 g}{m_2}$

1. $(c) \Rightarrow \ddot{y}_3 = \dfrac{m_3 g - 2T}{m_3}$

2. $(d) \Rightarrow 2\ddot{y}_3 - \dfrac{T - \mu m_1 g}{m_1} - \ddot{x}_2 = 0 \qquad (d')$

2. $(d') \Rightarrow 2\ddot{y}_3 - \dfrac{T - \mu m_1 g}{m_1} - \dfrac{T - \mu m_2 g}{m_2} = 0 \qquad (d'')$

2. $(d'') \Rightarrow 2\dfrac{m_3 g - 2T}{m_3} - \dfrac{T - \mu m_1 g}{m_1} - \dfrac{T - \mu m_2 g}{m_2} = 0 \qquad (d''')$

Now factor T in (d''') and solve for T:

$$\Longrightarrow \quad -T\Big(\frac{4}{m_3} + \frac{1}{m_1} + \frac{1}{m_2}\Big) + 2\mu g + 2g = 0 \qquad\qquad \Longrightarrow \quad T = \frac{2g(\mu + 1)}{\frac{4}{m_3} + \frac{1}{m_1} + \frac{1}{m_2}}$$

Find $\ddot{x}_1$, $\ddot{x}_2$, and $\ddot{y}_3$ by substituting T into (a), (b), and (c) [step 4].

Your job: (a) in Mathematica, Solve[] the above system; (b) also in Mathematica reproduce the intermediate (primed) equations above; (c) solve the systems below *by hand* (and then maybe test the reliability of Mathematica by seeing if it gets the same thing).

Practice Problem A: Solve for $\ddot{x}_2$ and T. [Note: These are just made-up equations!]

$$2T - \mu m_1 g = m_1 \ddot{x}_1 \qquad (a) \qquad\qquad m_2 g - T = m_2 \ddot{y}_2 \qquad (c)$$

$$T - \mu m_2 g = m_2 \ddot{x}_2 \qquad (b) \qquad\qquad \ddot{y}_2 - 2\ddot{x}_1 - \ddot{x}_2 = 0 \qquad (d)$$

Practice Problem B: Solve for x, y, and z. Make sure to *check* your answer by substituting back into the original equations!

$$3x + 2y - z = -2 \quad (a) \qquad\qquad x - y + 2z = 3 \quad (b) \qquad\qquad 2x + 3y - z = 1 \quad (c)$$