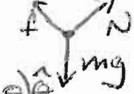
1. 
A bead slides with force diagram  which we express in 
as $\vec{F} = (N - mg\cos\theta)\hat{r} + (-f + mg\sin\theta)\hat{\theta}$

that's the direction θ increases in

Equating with $m\vec{a} = (0 - R\dot{\theta}^2)m\hat{r} + (2\cdot 0 + R\ddot{\theta})m\hat{\theta}$ we learn that
$N = mg\cos\theta - R\dot{\theta}^2 m$ and $f = mg\sin\theta - R\ddot{\theta}m$. Then since we are sliding,
$f = \mu N$ reads $g\sin\theta - R\ddot{\theta} = \mu(g\cos\theta - R\dot{\theta}^2)$ which is a really nasty differential equation.
If $\mu = 0$ we at least know we conserve energy, so if we start with $E_0 = \frac{1}{2}mv_0^2 + mgR$
then at θ we have $E = \frac{1}{2}mv^2 + R\cos\theta \implies v^2 = v_0^2 + 2gR(1-\cos\theta)$. Expressing $v = R\dot{\theta}$ here,
we then know that whenever we get to θ, the normal force will be $N = mg\cos\theta - \frac{mv^2}{R}$
$= mg(3\cos\theta - 2) - mv_0^2/R$

2. If $\dot{v} = -\alpha v^3$, then the units of the input parameter α are

$$[\alpha] = [\dot{v}]/[v^3] = (L/T^2)/(L/T)^3 = T/L^2$$

Trying the guess $v_A = Ae^{-\gamma t}$ we have $\dot{v}_A = -\gamma A e^{-\gamma t} \overset{?}{=} -\alpha(v_A)^3 = -\alpha A^3 e^{-3\gamma t}$
Since the t dependence doesn't match (unless $A=0$, which is ruled out) this one doesn't work.
Trying $v_B = B(1+\beta t)^n \implies \dot{v}_B = nB(1+\beta t)^{n-1}\beta \overset{?}{=} -\alpha(v_B)^3 = -\alpha B^3(1+\beta t)^{3n}$
The t dependence will match if $(n-1) = 3n \implies n = -\frac{1}{2}$. Then we need to choose $n\beta B = -\alpha B^3$,
where $v_0 = v_B(t=0) = B \implies \beta = 2\alpha v_0^2$, or $v(t) = v_0/\sqrt{1+2\alpha v_0^2 t}$

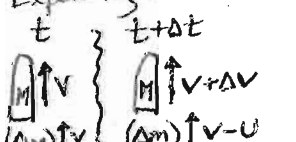
note the units work out so we can't have gone too far wrong

3. 
In the given initial state, $\vec{P}_{tot} = mv_0\hat{x} + mv_0\hat{y}$ so $\vec{V}_{CM} = \frac{\vec{P}}{M} = \frac{1}{2}v_0(\hat{x}+\hat{y})$
The velocity of one particle is $\vec{V}_{1CM} = v_0\hat{x} - \vec{V}_{CM} = \frac{1}{2}v_0(\hat{y}-\hat{x})$, with speed $|\vec{V}_{1CM}| = \frac{v_0}{\sqrt{2}}$
In an inelastic collision the final state has one object of mass $2m$ moving at $\vec{V}_{CM}$,
so the energy lost is: $(\frac{1}{2}mv_0^2 + \frac{1}{2}mv_0^2) - \frac{1}{2}(2m)|\vec{V}_{CM}|^2 = \frac{1}{2}mv_0^2$ (BTW this is also $K_{initial}$ in the CM)


In the elastic case we have three conservation relations:
$mv_0 = v\frac{\sqrt{3}}{2}m + mU_x$ and $2\cdot\frac{1}{2}mv_0^2 = \frac{1}{2}m(U^2 + v^2)$
$mv_0 = mv/2 + mU_y$ $\qquad U_x^2 + U_y^2$

Solving for $U_x = v_0 - v\frac{\sqrt{3}}{2}$ and $U_y = v_0 - \frac{v}{2}$ the K relation becomes $2v_0^2 = (v_0 - v\frac{\sqrt{3}}{2})^2 + (v_0 - \frac{v}{2})^2 + v^2$
Expanding and combining, $0 = 2v^2 - (1+\sqrt{3})v_0 v \implies$ either $v=0$ or $v = \frac{1}{2}(1+\sqrt{3})v_0$

not really "flying off" — the one we want

4. 
Between generic time t and $t+\Delta t$, a standard issue rocket shoots a small piece of fuel $\Delta m = b\Delta t$ out the exhaust, changing its velocity from v to $v-U$. The change in momentum is: $\Delta p = M(v+\Delta v) + \Delta m(v-U) - (M+\Delta m)v$
$= M\Delta v - (\Delta m)U$

Dividing by Δt etc. we learn $F_{ext} = -Mg = \frac{dp}{dt} = M\dot{v} - bU$

or... $\dot{v} = -g + \frac{bU}{M}$ where $M \equiv M_0 - bt$.

Note that to liftoff we need $\dot{v} > 0$, and so $b > \frac{M_0 g}{U}$. If this is satisfied, then $\dot{v}$ is always positive, since M only decreases from M_0.

And integrating, $v(t) = \int_0^t \dot{v}(t')dt' = \int_0^t \left(-g + \frac{bU}{M_0 - bt'}\right)dt' = -gt + U\log\left[\frac{M_0}{M_0 - bt}\right]$