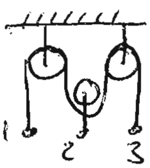
Phys 261 Midterm #1 — Solution

1. (a) Given that the constant K appears in the force law $F = -\frac{Km}{r^2}$, we find the units of K to be: $[K] = [F]\cdot[r^2/m] = (M\cdot\frac{L}{T^2})\cdot L^2/M = L^3/T^2$

(b) Then if K also appears in some relation for the period of an orbit (along with some m & R), we write $T = \# K^a m^b R^c \Rightarrow [T] = (L^3/T^2)^a M^b L^c = T^{-2a} M^b L^{3a-c} \stackrel{!}{=} T^1 M^0 L^0$

So to get the right units, $T = \# K^{-1/2} R^{3/2}$, and we've just obtained one of Kepler's laws.

2. Here are some proposed answers for T in the machine at left.

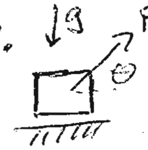


(a) $T = 4g\,\frac{m_1m_2m_3}{m_2m_3+4m_1+m_2}$ oops this one has mismatched units in the denominator

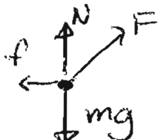
(b) $T = 4g\,\frac{m_1m_2m_3}{m_2m_3-4m_1m_3+m_1m_2}$ oops this one has $T\to\infty$ for perfectly ordinary combinations of masses, e.g. $m_1=m_3=m$, $m_2=2m$

(c) $T = 4g\,\frac{m_1m_2+m_2m_3+m_1m_3}{m_1+2m_2+m_3}$ when any of the three masses vanishes, we expect $T\to0$ (since otherwise the light mass would accelerate at ∞)
This expression fails: e.g. when $m_1\to0$ it gives $T = \frac{m_2m_3}{2m_2+m_3}4g$

3.



(a) Force diagram:



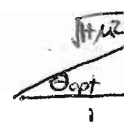
static $\Rightarrow$ $0 \stackrel{!}{=} -f + F\cos\theta$ $\Rightarrow f = F\cos\theta$

$0 \stackrel{!}{=} N + F\sin\theta - mg$ $\quad N = mg - F\sin\theta$

(b) To get the block to slide we need an F large enough that

$f = \mu N \Rightarrow (F\cos\theta) = \mu(mg - F\sin\theta) \Rightarrow F = \frac{\mu mg}{\cos\theta + \mu\sin\theta}$

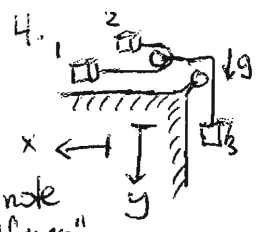
(c) So it takes the least effort to pull at the angle which makes this denominator the largest: $0 \stackrel{!}{=} \frac{d}{d\theta}[\cos\theta + \mu\sin\theta] \Rightarrow \tan\theta_{opt} = \mu$



Referring to the triangle we see that for that θ_{opt}, $\cos = \frac{1}{\sqrt{1+\mu^2}}$ & $\sin = \frac{\mu}{\sqrt{1+\mu^2}}$

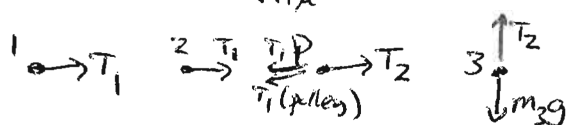
So $F(\theta_{opt}) = \mu mg\cdot\sqrt{1+\mu^2}/(1+\mu^2) = mg\,\frac{\mu}{\sqrt{1+\mu^2}}$

4.



note "funny" coordinates

(a) Force diagrams:



for our choice of y

(b) "checks" for $\ddot y_3$:
- if $m_3\to\infty$ we expect it "wins" and just falls, i.e. $\ddot y_3 = +g$
- or if $m_3\to0$ it just hangs there, i.e. $\ddot y_3\to0$
- or if one of m_1 or $m_2\to0$, the upper pulley slides freely & $\ddot y_3\to g$

(c) Newton's F=ma's:

$m_1\ddot x_1 = -T_1 \qquad m_2\ddot x_2 = -T_1 \qquad 0 = T_2 - 2T_1 \qquad m_3\ddot y_3 = m_3g - T_2$

(d) Rope constraints:

$$\left.\begin{aligned} l_1 &= (x_1-x_p)+\pi R_1+(x_2-x_p) &\Rightarrow 0 &= \ddot x_1+\ddot x_2-2\ddot x_p \\ l_2 &= x_p+\pi R_2/2+y_3 &\Rightarrow 0 &= \ddot x_p+\ddot y_3 \end{aligned}\right\} \Rightarrow \ddot x_1+\ddot x_2+2\ddot y_3 = 0$$

(e)
One path through the algebra: use F=ma's to express $\ddot x_1 = -T_1/m_1$, $\ddot x_2 = -T_1/m_2$, $\ddot y_3 = g - 2T_1/m_3$

and plug these into the constraint: $(-\frac{T_1}{m_1})+(-\frac{T_1}{m_2})+2(g-\frac{2T_1}{m_3}) = 0 \Rightarrow T_1 = 2g/(\frac{1}{m_1}+\frac{1}{m_2}+\frac{4}{m_3})$

Then $\ddot y_3 = g - \frac{2T_1}{m_3} = g\left[1-\frac{4}{m_3}\cdot(\frac{1}{m_1}+\frac{1}{m_2}+\frac{4}{m_3})^{-1}\right] = g/(1+\frac{4m_1m_2}{m_3(m_1+m_2)})$

And indeed if $m_1\to0$ or $m_2\to0$ or $m_3\to\infty$, this term goes away, leaving $\ddot y_1\to g$
while if $m_3\to0$ that term blows up, leaving $\ddot y_1\to0$ as expected.