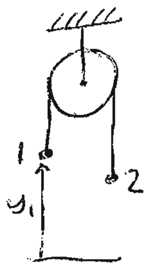
261 Practice Midterm

1. (*20 points*) Two masses m_1 and m_2 are attached by a string and draped over a massless frictionless pulley, just as on the course webpage. Suppose someone else (**not you**) is asked to solve for the vertical acceleration of mass one, $\ddot{y}_1$, where y_1 increases up the page.



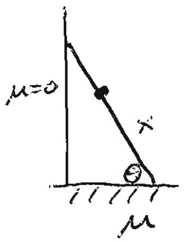
   (a) There are three extreme cases where you can "check" the answer. What do you expect for $\ddot{y}_1$ (1) when $m_1 \ll m_2$, (2) when $m_1 = m_2$ and (3) $m_1 \gg m_2$? (Note: our convention is that $g > 0$, so "free-fall" is an acceleration of $y_1 = -g$.)

   (b) Does this expression pass the checks: $\ddot{y}_1 = -\frac{(m_1-m_2)^2}{m_1^2+m_2^2}g$?

   (c) What about this one: $\ddot{y}_1 = -\frac{(m_1+m_2)}{m_1-m_2}g$?

   (d) And this one: $\ddot{y}_1 = \tanh(\frac{m_2}{m_1} - \frac{m_1}{m_2})g$?

2. (*30 points*) Consider a *massless* ladder of length L leaning up against a frictionless wall at an angle θ relative to horizontal. The coefficient of friction between the ladder and the ground is μ. A person of mass m wishes to stand at a distance x up the ladder.



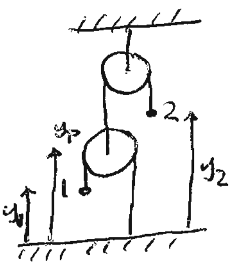
   (a) Draw the force diagram.

   (b) Assuming the parameters are such that the ladder will not move or spin, write down the three relations which let you determine all of the contact forces.

   (c) Determine those forces.

   (d) What is the largest x can become before the ladder slips?

3. (*50 points*) Consider the system of two masses, two ropes and two pulleys as drawn. To get you going, I've indicated the coordinates to use, including one for the movable pulley.



   (a) Before writing any equations, what do you expect for $\ddot{y}_1$ in the limit $m_1 \gg m_2$? And what of $\ddot{y}_2$ when $m_1 \ll m_2$?

   (b) Write Newton's 2nd law for each subsystem. Note that the movable pulley is of the usual massless variety, so its $F = ma$ relation says the forces have to balance to zero (or else the pulley would accelerate at an unreasonably rate.)

   (c) Since there are two ropes, there are two contraint relations of the form $\ell_{rope} = \cdots$. Write these relations down, inventing names for various constants as you like. What are the resulting relationships between accelerations?

   (d) Find the accelerations $\ddot{y}_1$ and $\ddot{y}_2$, and "check" against the expectations of part (a).