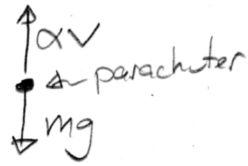
Physics 261 Math Session

As baseball season ends, let's start today with an item most folks haven't had time to get to: the homerun ball.

- DSolve to find $x(t)$ and $y(t)$ given that:

$$\ddot{x} = -\alpha\dot{x} \qquad \ddot{y} = -g - \alpha\dot{y} \qquad x(0) = y(0) = 0 \qquad \dot{x}(0) = V_0\cos\theta \quad \dot{y}(0) = V_0\sin\theta$$

- Choose "reasonable" numerical values, e.g. $g = 10$ (m/s²).
  - you know that pitchers throw at $v \sim 100$ mph, so take the speed of the batted ball is ~ 100 mph
  - you know that parachuters reach a "terminal velocity" of ~ 100 mph (actually 120), so choose α appropriately



- Make a ParametricPlot of the trajectory
- How far does the ball go? How high? How long in the air?
- Finally make a movie.
  - Here is how to draw a ball: ListPlot[{{x,y}}, PlotStyle→{PointSize[.03]}]
  - Add the PlotRange→{{x_{min}, x_{max}}, {y_{min}, y_{max}}} option to fix the viewport
  - Here is how to make it move: Animate[ListPlot[...], {t, 0, 5}]
  - And here is how to see the path at the same time:

    Animate[Show[p1, ListPlot[...]], {t, 0, 5}]

    where p1 = ParametricPlot[...]

In the mass-on-a-string problem this week we have:

$$\vec{F} = -T\hat{r} \longrightarrow 0 = a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$$

Given that $r(t) = r_0 - v_0 t$, DSolve for $\theta(t)$ with initial conditions $\theta(0) = 0$, $\dot{\theta}(0) = \omega_0$. Choosing numerical values for the parameters (in appropriate units, $v_0 = \frac{1}{10}$ m/s, $\omega_0 = 1$ radian/s, $r_0 = 1$m), ParametricPlot the trajectory. PRINT THIS CELL* and hand in w/the homework. Optionally, decorate the curve with a ListPlot[] of dots marking the location every so often.

note you will need to form ($r\cos\theta$, $r\sin\theta$) since ParametricPlot uses (x,y)

* more precisely — wait and print

Next consider the bead on a rod problem. Force diagram:

The frictionless rod is constrained to rotate with $\theta(t) = \omega_0 t$. To find $r(t)$, examine the $F_r = ma_r$ equ: since $F_r = 0$, we know that $\ddot{r} - \omega_0^2 r = 0$

- DSolve for $r(t)$ with initial conditions $r(0) = r_0$ $\dot{r}(0) = 0$
- again choose parameters, ParametricPlot and <u>PRINT</u>

Lastly try finding the trajectories for the funny forces on the hwk:

$F_\theta = m\dot{r}\dot{\theta}$ and $F_\theta = 3m\dot{r}\dot{\theta}$

For the latter you are eventually supposed to show that:

$\dot{r} = \sqrt{Ar^4 + B}$ where A & B depend on initial conditions.

Supposing $A=1$, $B=1$, how much time does it take to get from radius $r=1$ to $r=2$? How much time to get from $r=1$ to $r=\infty$?!

PRINT the answers and include w/ hwk.