

Physics 416 Take Home Final Exam

Due by June 6, 2007 4 PM**The exam is to be given to Michael Wells (PRB 3049)****Exam Rules:**

a) Work alone. You are free to consult with any library book(s) you wish. If you do use references other than the notes and text please list them.

b) Where possible show your work. Do not just write down the answer (especially if you hope to get partial credit)!

1) Short answer questions, each worth 5 points.

a) What is the difference between precision and accuracy?

b) What is the difference between a statistical error and a systematic error?

c) What is a “type 1” error?

d) Define the mean, mode, and median of a probability distribution.

e) Assume I have 10 data points which I believe are described by a Poisson distribution. If I use the 10 data points to calculate the average of the Poisson how many degrees of freedom do I have left?

f) Can the probability for an event to occur be negative?

g) True or False: The variance is a measure of the spread of the data around the mean.

h) Under what assumption(s) does the maximum likelihood method, chi-square fitting, and method of least squares all give the same results?

i) Under what conditions does a binomial distribution “turn into” a Poisson distribution?

j) True or False: A good “rule of thumb” is that a chi-squared per degree of freedom of one is an acceptable fit.

2) Binomial and Poisson probability distribution.

a) 25 points. Show that the mean of binomial distribution for N trials where the probability for success= p is given by Np .

b) 25 points: Show that the variance of a Poisson probability distribution is equal to its mean.

c) 20 points: Make a table of the binomial probabilities for $N=6$, $p=0.5$. Make a table of the Poisson probabilities for $0 \leq n \leq 6$ assuming the same mean as the $N=6$, $p=0.5$ binomial.

3) Given the probability distribution function:

$$p(x,A)=A(1-x^2) \text{ with } -1 \leq x \leq 1$$

a) 10 pts: Find the normalization constant A .

b) 10 pts: Calculate the variance of x over the interval $-1 \leq x \leq 1$.

c) 10 pts: What is the most probable value of x in the interval $-1 \leq x \leq 1$?

d) 5 pts: What's the probability for x to be in the range $-0.2 \leq x \leq 0.1$?

4) Central Limit Theorem Problem:

a) 5 pts: Describe or state the Central Limit Theorem.

b) 5 pts Under what conditions is the CLT valid?

c)) 30 pts: Assume that the probability of producing N particles in a single high energy proton collision can be described by the following probability distribution:

$$P(N) = \frac{16^N e^{-16}}{N!}$$

If the total number of produced particles is counted after 100 collisions what is the probability that the total number of particles will be in the range [1600, 1700]? Hint: Which probability distribution is $P(N)$?

5) 30 points: Taylor Problem 3.38, page 88.

6) 30 pts: Taylor Problem 5.36 page 161.

7) The lifetime of ^{137}Ba was measured by several students in our class. Four independent results are given:

(3.70±0.05) minutes

(3.72±0.10) minutes

(4.72±0.15) minutes

(3.80±0.10) minutes

Assume Gaussian statistics apply here.

a) 10pts: What is the best estimate of the lifetime if we combine the results of the four experiments?

b) 10 pts: What is the variance of the lifetime calculated in a)?

c) 5 pts: Which of the four measurements is the most precise and which is the least precise?

8) A Maximum Likelihood Method Problem.

a) 5pts: State or describe the Maximum Likelihood Method.

Assume that the probability distribution function, $p(x,\alpha)$, to find two quarks separated by a distance x (in units of 10^{-15} m) depends on an unknown constant α ,

$$p(x, \alpha) = \alpha x^{\alpha-1} \text{ with } 0 \leq x \leq 1 \text{ and } 0 < \alpha < \infty$$

and we have five measurements of x ($x_1=0.45$, $x_2=0.68$, $x_3=0.36$, $x_4=0.87$, $x_5=0.54$).

b) 10pts: Write down the likelihood function for this problem.

c) 10pts: Use the Maximum Likelihood Method to derive an expression for α .

d) 10pts: Use the Maximum Likelihood Method to calculate a value for α .

Note: if necessary, part d) can be done without part c).

9) Five measurements of the amount of a certain radioactive material vs time are given below. Assume the sample decays exponentially, i.e.

$$N(t) = N_0 e^{-t/\tau}$$

with τ the lifetime and N_0 the amount of material present at $t=0$.

Time (minutes)	N(t)
10	409
20	304
30	260
40	192
50	170

a) 30 pts: Find τ and N_0 by performing an unweighted least squares fit to:

$$\ln[N(t)] = \ln[N_0] - t/\tau$$

where $\ln[x]$ is the natural log of x .

b) 20 pts: Calculate the χ^2 per degree of freedom of the fit. What is the probability to obtain a χ^2 per degree of freedom greater than or equal to this? Is this an acceptable fit?

Note: Assume that the number in each bin follows Poisson statistics so the variance for each bin is given by the number of counts in the bin.

10) A Confidence Level Problem.

An experiment performed a search for evidence of proton decay ($p \rightarrow \gamma e^+$) using a large vat of water. Use the Poisson probability distribution to answer parts a) and b).

a) 10 points: When the experiment ended, it had no examples of proton decay. Calculate the 95% confidence level for the mean number of proton decays occurring during the experiment.

b) 15 pts: Suppose the experiment had two candidates for proton decay. What would the 95% confidence level for the mean number of proton decays occurring during the experiment be now?

11) For the following γ -ray spectrum assume that the photopeak has energy = 660 keV.

a) 15 pts: Calculate the energy of the Compton edge and note where it is located on the spectrum.

b) 15 pts: Calculate the energy of the 180° backscatter peak and note where it is located on the spectrum.

c) 10 pts: Using your results from a) and b) estimate the energy of the “mystery peak”.

