

Problem Set 3
Due May 1, 2007

1) The probability distribution function (*pdf*) (often called a Maxwellian Distribution) that describes the speed ($w = \text{magnitude of velocity}$) of molecules in an ideal gas is given by:

$$p(w) = \frac{4}{\sqrt{\pi}} \left(\frac{m}{2kT} \right)^{3/2} w^2 e^{-\frac{mw^2}{2kT}}$$

Here $m = \text{mass of the molecule}$, $k = \text{Boltzmann's constant}$, and $T = \text{temperature}$.

a) Show this is a properly normalized *pdf*.

In terms of k , T , and m :

b) Find the most probable speed.

c) Find the average speed.

d) Find the variance of the speed.

2) A Central Limit Theorem problem. When a batch of a certain type of superconductor is produced the amount of an impurity in the material (considered to be a random variable) must be tightly controlled. The amount of the impurity in a batch of superconductor has a mean of 4 micrograms and a standard deviation of 1.5 micrograms. If 100 independent batches (assume each batch weighs 200 micrograms) of the superconductor material are produced what is the (approximate) probability for the total amount of the impurity in the 100 batch sample being more than 425 micrograms?

Propagation of Errors (Uncertainties) Problems:

3) Taylor P3.22, page 83.

4) Taylor P3.24, page 83.

5) Taylor, problem 3.28, page 85.

6) Taylor P3.46, page 90.

Do 7a) OR 7b). (Actually, you can do both but you will only receive credit for one part.)

7a) Write a computer program and use the monte carlo method to determine the uncertainty and compare your answer with the one from propagation of errors. To use the monte carlo method: 1) generate 10^6 values of x and y assuming that x and y are from Gaussians with their given mean and standard deviation, 2) for each (x, y) pair calculate q , 3) make a histogram of your 10^6 q 's, 4) from your histogram determine the value of q , $q_{\pm\sigma}$, where 32% of the q 's are between $[q_{\text{average}}, q_{+\sigma}]$ and 32% are between $[q_{-\sigma}, q_{\text{average}}]$.

7b) Taylor, problem 3.50, page 91. Remember to use radians and not degrees!

8) The decay of an unstable particle is given by the following probability distribution function in terms of the decay time (t) and the particle's lifetime (λ).

$$p(t, \lambda) = \frac{e^{-\frac{t}{\lambda}}}{\lambda}$$

Four measurements of t ($t_1 = 3.1$ sec, $t_2 = 6.9$ sec, $t_3 = 4.1$ sec, $t_4 = 2.9$ sec) are made.

a) Write down the likelihood function (L) for this problem.

b) Use the Maximum Likelihood Method to calculate the value of λ for this data set.