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Problem Set 2 Physics 416
Due April 19 2007

1) Assuming a Gaussian probability distribution answer the following questions
(Use the tables in *Taylor Appendix A and/or B)*:
  a) What is the probability of a value lying $\leq 1.5\sigma$ below the mean?
  b) What is the probability of a value lying $\geq 1.5\sigma$ above the mean?
  c) What is the probability of a value lying more than 1.5σ from the mean?
  d) What is the probability of a value, y, lying in the range $(\mu-2.5\sigma) \leq y \leq (\mu-\sigma)$
  e) What is the probability of a value, y, lying in the range $(\mu+\sigma) \leq y \leq (\mu+2\sigma)$

For this problem μ is the mean of the gaussian and σ is its standard deviation.

2) Taylor, Problem 5.12, page 156.

3) The sun emits an enormous number of neutrinos. Assume that 10^6 solar neutrinos uniformly pass through a square 1 m on a side each μsec. Inside the square is a neutrino detector with area=1 mm^2. Assume Poisson statistics for this problem.
a) What is the average number of neutrinos going through the detector each μsec?
b) What is the probability that no neutrinos go through the detector in a μsec?
c) What is the probability that ≥ 2 neutrinos go through the particle detector in a μsec?
d) How big should the detector be (in mm^2) if we want ≥ 2 particles per μsec to pass through the detector with a probability of 95%?

4) Suppose a missile defense system destroys an incoming missile 90% of the time.
a)If an evil country launches 20 missiles what is the probability that the missile defense system will destroy all of the incoming missiles?
b) How many missiles have to be launched to have a 50% chance of at least one missile making it through the defense system? Note: this problem can be solved using either binomial or Poisson statistics.

5) According to quantum mechanics, the position (x) of a particle in a one dimensional box with dimensions $-L/2 \leq x \leq L/2$ (L constant) can be described by the following probability distribution function p(x):

$$p(x)=A\cos^2[\pi x/L] \text{ for } (-L/2 \leq x \leq L/2), \text{ and 0 for all other x.}$$

a) Find the normalization constant A in terms of L.
b) Find the mean, mode, and median position of the particle in the box.
c) Show that the variance of x (σ^2) is given by:

$$\sigma^2 = \left(\frac{L}{\pi}\right)^2 \frac{\pi^2 - 6}{12}$$

d) What is the probability of finding the particle in the region: $-L/4 \leq x \leq L/4$?

6) Suppose 100 six sided dice are tossed. Assume that the faces are labeled by the numbers one through six. Let Y_i be the number showing on the *i*th (i=1 to 100) die. Use the Central Limit Theorem to estimate the probability that the sum of the Y_i 's exceeds 325.

7) The following probability distribution function (*pdf*), *p*(x), is commonly used in nuclear and high energy physics:

$$p(x) = \frac{\delta}{(x-\alpha)^2 + \beta^2}$$

In the above equation α, β, δ are constants and $(-\infty < x < \infty)$.
  a) Find the value of δ needed to normalize this *pdf*.
  b) Find the mean of this *pdf*.
  c) Find the variance of this *pdf*.
  d) Does this *pdf* satisfy the conditions of the Central Limit Theorem? Explain your answer.