Chapter 29: Magnetic Fields

Magnetism has been known as early as 800BC when people realized that certain stones could be used to attract bits of iron.

Experiments using magnets have shown the following:
1) EVERY magnet has two “poles” which we refer to as north and south. These poles act in a way similar to electric charge, north and south attract, north repels north, south repels south.

An important difference between electric charges and magnetic poles is that poles are ALWAYS found in pairs (N,S) while single electric charges (positive or negative) can be isolated. For example, if you cut a bar magnet in half each piece will have a N and S pole!

2) The forces between magnets are similar to those between electric charges in that the magnitude of the force varies inversely with the square of the distance between them. As we shall see, calculating the direction of the magnetic force is more complicated than the electric field case.

In the 1800’s many experiments illustrated that electricity and magnetism were connected.
1) An electric current can exert a force on a magnet.
2) A magnet was shown to exert a force on a current carrying conductor.
3) An electric current can be induced by moving a magnet near it or by changing the current in a nearby circuit.

In 1865 J. C. Maxwell writes the equations that unify electricity and magnetism. (only takes 4 equations!)
The Magnetic Field and Force

In our study of electricity we defined the electric field (E) by studying the force on an electric charge (q):

\[ \vec{F} = q\vec{E} \]

Since there is no such thing as a single magnetic pole ("monopole") we cannot write an analogous equation for the relationship between the magnetic field and its force. From experiments using electrically charged particles moving in regions near a magnet we find:

1) The magnitude of the magnetic force is proportional to electric charge of the particle.
   A positively charged particle and negatively charged particle experience forces in opposite directions.
2) The magnitude of the force is proportional to the magnitude of the particle’s velocity (v).
3) The magnitude of the force is proportional to the magnitude of the magnetic field (B).
4) The magnitude of the force is proportional to \( \sin(\theta) \) where \( \theta \) is the angle between the velocity and magnetic field vectors.
5) The direction of the force is always perpendicular to both B and v.

Items 1)-5) can be summarized in one equation:

\[ \vec{F} = q\vec{v} \times \vec{B} \]

The magnetic field (B) is a VECTOR quantity.
The magnetic force is given by a “cross product” of vectors.
The magnitude of the magnetic force is:

\[ |\vec{F}| = qvB \sin \theta \]

The units of B are Tesla=kg/C/s.

You have seen the cross product before if you have studied torques.
The Magnetic Force

Calculating the direction of the magnetic force is more challenging than calculating the direction of the electric force.

To calculate the direction of the electric force we need to know 2 quantities:
- sign of the electric charge
- direction of the electric field

To calculate the direction of the magnetic force we need to know 3 quantities:
- sign of the electric charge
- direction of the charged particle’s velocity
- direction of the magnetic field

The direction of the magnetic force is perpendicular to the velocity and magnetic field.

For positive charge moving in a magnetic field we can find the direction of the force using THE RIGHT HAND RULE (HRW p660)

The right hand rule giving the direction (thumb) of $\vec{v} \times \vec{B}$

The right hand rule gives the direction (thumb) of $\vec{F}$ for a positively charge particle

If the charge is negative then the direction of the force is opposite to the direction of $\vec{v} \times \vec{B}$
The Magnetic Force continued

Examples of using the right hand rule to calculate the direction of the force. In all examples the velocity and magnetic field vectors lie in the plane of this page.

If the charge is positive then the direction of F is out of the page.
If the charge is negative then the direction of F is into the page.

If the charge is positive then the direction of F is into the page.
If the charge is negative then the direction of F is out of the page.

In both cases there is no magnetic force since sin(θ)=0!
Magnetic Field Lines

Just as the case with the electric field we can draw field lines to help visualize the magnetic field. Here are the rules:
1) For a magnet the field lines exit from the north pole and enter the south pole.
2) Magnetic field lines are always closed loops since there is no such thing as a single magnetic charge.
3) The field is large where the field lines are close together and where the lines are farther apart the field is weaker.
4) The direction of the magnetic field is tangent to the field line at that point.

Examples showing magnetic field lines for a bar, horseshoe, and C-shaped magnet.
Motion of a charged particle in a magnetic field

A charged particle moving in a B-field experiences a force that is perpendicular to its direction of motion. The acceleration due to the B-field is also perpendicular the particles velocity. The net result of the acceleration is that the particle bends as it moves through the B-field. The magnitude of the velocity does not change, only the direction of the velocity changes. This is exactly the same as the case for a particle that undergoes uniform circular motion.

One of the more interesting situations is when we have both electric and magnetic forces. Let’s add a uniform electric field to the above figure. Assume we have a positively charged particle.

The force on the charge is: \[ \vec{F} = q\vec{v} \times \vec{B} + q\vec{E} \]

The magnitude of the force is: \( F=qvB-qE \)

The particle will pass undeflected if we choose the speed \( v: v=E/B \)

Thus we can create a *velocity selector* by choosing \( E \) and \( B \).

This leads to a procedure that allows us to measure mass to charge ratio of a particle!
Measurement of the Mass to Charge Ratio

Using crossed electric and magnetic fields we can learn about the mass to charge ratio of a particle. This led to the discovery of the electron by J. J. Thomson in 1897.

In sample problem 23-4 HRW, p534, calculates the deflection (y) due to \( F_E \).

\[
\vec{F} = q\vec{E} \Rightarrow a_x = \frac{qE}{m}
\]

Before entering the region of electric field \( v_x = 0 \), \( v_y = v \). Therefore:

\[
x = x_0 + v_x t + \frac{a_x t^2}{2} = \frac{qEt^2}{2m}
\]

We can eliminate the time using: \( t = \frac{L}{v} \). Therefore the deflection is:

\[
x = \frac{qEL^2}{2mv^2}
\]

We now have a way to measure \( m/q \! \!:

1) Get a beam of particles with a unique speed and put them through a region of known length (L), electric field (E), and measure their deflection (x).
2) Measure the speed of the particle (v) by turning on a magnetic field perpendicular to E (out of page here) and tune B until the deflection x is zero.
3) The mass to charge ratio is:

\[
\frac{m}{q} = \frac{B^2L^2}{2xE}
\]

4) Collect Nobel Prize (Thomson, 1906).
Magnetic Force on Current Carrying Wire

If a wire carrying a current is placed in a magnetic field it will experience a force if the B-field is perpendicular to the wire.

Consider the charge carriers moving through the wire. They move with the speed of the drift velocity $v_d$. The force on these charges is:

$$F = qv_dB\sin \theta,$$

($\theta$= angle between drift velocity and B).

The amount of charge that passes through a cross section of the wire at time $t$ is:

$$q = It \quad (I=\text{current in wire})$$

In terms of the drift velocity we can re-write $q$ as:

$$q = I(L/v_d)$$

The force on these charges can be written as:

$$F = qv_dB\sin \theta = I(L/v_d)v_dB\sin \theta = ILB\sin \theta$$

In vector notation we have:

$$\vec{F} = I\vec{L} \times \vec{B}$$

If the wire is not straight or if the B-field is not uniform we can calculate the force from:

$$d\vec{F} = Id\vec{L} \times \vec{B} \Rightarrow \vec{F} = I \int d\vec{L} \times \vec{B}$$

**Example:** How large a magnetic field do we need to suspend a copper wire (mass $m$, length $L$) with current $I$ running through it?

In order to suspend the wire we must counter act gravity, so we require: $mg = IBL$ or $B = (mg)/(IL)$.

For copper $m/L$ is a constant (its mass density) = $46.6 \times 10^{-3}$ kg/m, and using $g = 9.8 \text{ m/s}^2$ we have:

$$B = 0.46/I \text{ (Tesla)}$$

If a wire carries 100A then $B \approx 5 \times 10^{-3} \text{ T}$. (This is about 50X the earth’s magnetic field)
Circular Motion in a magnetic Field

The magnetic force on a charged particle is always perpendicular to its velocity. You have probably seen a similar situation in a mechanics course (P131) when uniform circular motion was discussed. For example, putting a ball at the end of a rope and then whirling the ball around in a circle. To the right is a figure (HRW Ch 4-18) showing the velocity and acceleration vectors for a particle undergoing uniform circular motion. Like the case with the magnetic field the acceleration is perpendicular to the velocity.

A charged particle in a uniform (=constant) magnetic field will undergo circular motion. In fact, if the region of the magnetic field is large enough, the particle will be trapped in the space orbiting forever in a circle!

Let’s calculate the radius (r) of its orbit and how long it takes to complete one revolution. Since the motion is circular we have:

\[ F = qvB = m \frac{v^2}{r} \Rightarrow r = \frac{mv}{qB} \]

The time it takes for one complete revolution is the circumference of the circle divided by the particles speed.

\[ T = \frac{\text{distance}}{\text{speed}} = \frac{2\pi r}{v} = 2\pi \frac{1}{v} \frac{mv}{qB} = \frac{2\pi m}{qB} \]

The angular frequency (\( \omega \)) is \( 2\pi f \) and \( f=1/T \):

\[ \omega = 2\pi f = \frac{2\pi}{T} = \frac{qB}{m} \]

The frequency is independent of the speed of the particle (as long as the speed is much less than the speed of light)