Nonlocality

John Wear
The Ohio State University
Columbus, OH 43210
E-mail: wear.1@osu.edu

Abstract

A physical system exhibits *nonlocality* if it does not obey the principle of locality, the expectation that, if one part of a spatially dispersed system is influenced, the other parts of the system will not reflect that influence instantly. An example of a system that exhibits nonlocality is a pair of entangled particles that are physically separated in a space-like manner. Such a system is described, the hidden variables model for the behavior of this system is examined, and a refutation of this model based on Bell's inequality is offered.

1 Introduction

This paper provides an overview of the quantum phenomenon of nonlocality. We refer specifically to both thought and actual experiments that exhibit what Einstein referred to as "spooky action-at-a-distance" [1]. In an experiment on a system that enjoys nonlocality, a measurement on one part of the system affects another part of the system instantly, with no apparent mechanism for the propagation of the effect. This is a violation of the principle of locality, which insists that the effect propagate by a knowable physical mechanism, and therefore that the propagation be no faster than the speed of light.

The principle of locality is sometimes confused with *local realism*. To quote, "Local realism is the combination of the principle of locality with the assumption that all objects must objectively have their properties already before these properties are observed. Einstein liked to say that 'the Moon is out there' even when no one is observing it" [2].

We begin by describing a thought experiment involving quantum entanglement. One possible explanation for the strange behavior exhibited by such a system employs *hidden variables*. We examine the argument for hidden variables as given in EPR[1]. We then outline the refutation of this argument; the refutation employs *Bell's inequality*.

2 Quantum Entanglement

Consider the decay of a neutral pi meson, resulting in the creation of an electron and a positron,

$$\pi^0 \longrightarrow e^+ + e^-.$$

After decay, the electron and positron fly off in opposite directions along a line in space. If the neutral pi meson is at rest when it decays, then the total angular momentum of the system is zero, and the two resulting particles are in the singlet state[2],

$$|\psi\rangle = \frac{1}{\sqrt{2}}\{|\uparrow_-\downarrow_+\rangle - |\downarrow_-\uparrow_+\rangle\}. \tag{1}$$

Here and throughout, a state vector with a minus in the subscript will refer to the electron's state, and a state vector with a plus in the subscript will refer to the positron's state. With two particles in such a state, there is a 50-50 chance that a measurement of either particle's spin will yield spin up or spin down[3]. Then by conservation of angular momentum the other particle will need to be the opposite spin in order for the total angular momentum of the system to be zero.

Suppose we were somehow able to observe a neutral pi meson decay at a "near midpoint", a little closer to earth than Alpha Centauri. We want the positron or electron, it matters not which, to hit our laboratory on earth[4] at some time shortly before[5] the other particle is measured on

[1] EPR is common shorthand for the paper entitled *Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?* [3]. The paper was written by A. Einstein, B. Podolsky, and N. Rosen; ergo the shorthand EPR.

[2] Refer to the Appendix for a proof of why this is not just a game with notation.

[3] In this first version of the experiment, we will make things easy from the thought experiment perspective and assume that any pair of detectors are aligned, so that we measure along the $\hat{z}$ direction for both detectors.

[4] This is where the halfway point business is a little sticky. In practice, we need to take into account, at the moment of decay, the relative trajectories of the earth, our solar system, Alpha Centauri, Alpha Centauri's solar system... You get the idea: It is a mess.

[5] Although different observers might disagree about the order of measurement events, the order is fixed in the pion's rest frame. The important bit here is that the measurement of events on earth and Alpha Centauri enjoy a space-like separation.

Alpha Centauri. Suppose we measure the spin of our particle on earth to be spin-up. Then we can be assured that when the Centaurans measure the other particle, it will be spin-down. Not only do we know this, but we also know that the other particle, the one measured on Alpha Centauri, *instantly* "became" spin-down when its entangled partner was measured! Indeed, we could arrange the decay so that the Centaurans measure their particle's spin an arbitrarily short time after we had measured ours, so the spin of the Centaurans' particle must possess its determined spin state an arbitrarily short time after the Earthlings' measurement. Suppose not, and that instead the propagation of the singlet state collapse traveled at the speed of light. Then there would be, for up to roughly 3 earth years, a 50-50 chance that the Centaurans would measure their particle to have the same spin as our particle, which would give, 50% of the time over many such measurements, a violation of the conservation of angular momentum.

3 EPR

The entangled state thought experiment was first proposed, in a somewhat different form from that given above, in EPR [3]. But EPR explicitly *did not* accept the instantaneous collapse of the wave function. Instead, EPR posited a form of the hidden variable argument to explain the result that, it was assumed, would be obtained from a measurement of part of a physically disjoint entangled system[6].

The hidden variable argument holds that quantum theory is incomplete in that the statistical nature of measurements of quantum systems is not inherent but rather caused by insufficient knowledge of all the properties of the system. Were we to know all these properties, i.e. the hidden variables, then we could completely determine the evolution of a quantum state, much as we can completely determine the evolution of a classical system by knowing masses, relative initial kinetic and potential energies, initial positions, and so forth.

As we pointed out, the thought experiment used in EPR is somewhat different from our example. In our example, we measure spin along an aligned axis. The spin of either particle can be measured, and that measurement will not leave the spin along the same axis of the other particle

[6]There was no way in 1935 to actually carry out such an experiment in the laboratory. In fact, there was no way to carry out much more testable refinements of the experiment even in 1964, when Bell first addressed the problem [4].

undetermined; quite the opposite[7]. EPR employs two non-commuting measurements, of momentum and position. In fact, a piece of the argument in EPR referenced Bohr's *complementarity principle* to help demonstrate the incompleteness of quantum theory. In 1935, the year EPR was published, the complementarity principle was the dominant epistemology of quantum theory. The online version of the Encyclopedia Britannica gives the following definition: "Depending on the experimental arrangement, the behavior of such phenomena as light and electrons is sometimes wavelike and sometimes particle-like; i.e., such things have a wave-particle duality. It is impossible to observe both the wave and particle aspects simultaneously" [5].

Now we have enough background so that we can list a first-order approximate outline of EPR:

1. An assertion of local realism;
2. A claim that a physical theory either violate local realism or suffer incompleteness;
3. The use of an entangled-state thought experiment to show that a measurement of either one of a set of complementary properties of one particle in the entanglement would result in the other particle's corresponding property being defined;
4. Since local realism is assumed, the second particle must have had well-defined values for both of the complementary properties "all along";
5. Since there can be no violation of local realism, quantum theory must be incomplete.

More explanation is required. The claim in (2) is not stated this way in the paper. The violation of local realism is implicit in the inability to measure values for complementary attributes, (i.e. non-commuting operators), which is what EPR really gives as the choice in the dichotomy. Item (2) is also very "formulaic" from a logic standpoint[8].

Local realism, and in particular locality, are crucial to the argument in EPR. By assuming a separate reality for the two particles, and by assuming

[7] Later we will modify the experiment so that we can independently rotate the detectors' orientation relative to each other.

[8] The author enlisted the help of a colleague with formal training in logic but was unsuccessful in gaining more confidence in the logical structure of EPR's argument. Several authoritative sources, such as the Stanford Encyclopedia of Philosophy [6], criticize that structure for its opacity, but not its correctness.

that there is no interaction mechanism between them, the claim that the particles inherently possess all of the measured attributes from the start is compelling.

Obscured by the structure of the argument in EPR is a valid criticism of the complementarity principle, at least as it was understood at the time. Bohr himself was apparently bowled-over when he read EPR [7]. Bohr and many other quantum theorists at the time believed that complementarity arose because of the interaction between the measuring instrument and the object of measurement; this is the "little objects bumping into large measuring devices" model that Bohr relied on to give a physically graspable explanation for complementarity. EPR put *that* model in the grave. With some complaint about the validity of the wave function that EPR used, most readers accept the thought experiment. Whatever causes the wave function to behave as it does, the measurement instrument is not the cause. That is not to say that the instrument cannot be part of the system; it is just that the instrument measuring the particle on Earth obviously cannot affect the particle on Alpha Centauri via some sort of "bumping".

There is a rich history of the reaction to EPR, a history that includes Einstein, Bohr, Bohm, Born, Von Neumann, Schrödinger, and other famous physicists and philosophers. However, we will now fast-forward through all that, almost 30 years, to the work of the man who made so much clear: John S. Bell.

4 The Inequality

In 1964, John Bell showed that the hidden variables explanation of entangled particle behavior did not correlate with *thought* experimental results[9]. Before we outline the proof, we need to modify the thought experiment. We originally used detectors on earth and Alpha Centauri that were aligned along the $\hat{z}$ direction. Now we will allow each detector to point along independent unit vectors. The earth-Alpha Centauri scenario is no longer necessary; just call the detectors A and B. Detector A will point in the $\hat{a}$ direction, and detector B will point in the $\hat{b}$ direction. A result from quantum mechanics shows[10] that the average value of the product of the spins is given by,

[9] It was still the case in 1964 that no one had developed the experimental apparatus to test these scenarios.

[10] See the Appendix for a proof.

$$P(\hat{a}, \hat{b}) = -\hat{a} \cdot \hat{b}.$$

What John Bell showed was that any hidden variable theory will not accommodate the results predicted by quantum mechanics. The incompatibility is due to the above relation. We now outline a modified form of the proof that Bell gave[11].

Not only will we assume that each detector can be rotated independently; we will also assume that we can rotate the second detector before any subluminal message can arrive from the first detector. We assume that the probabilistic results we obtain for measurements in our system are due to some unknown variables involved in the system. Maybe electrons and positrons have a physical quantity called *stink* of which we are ignorant; but if we *did* know about it, we would definitely know what spin state we would get no matter what orientation our detectors had relative to each other. Of course, *stink* is just an unfortunate example. However, once we allow some one unknown, we could perhaps have any number of unknowns. Let λ represent this collection of unknowns; we claim λ to be a set of *random variables*[12].

When we measure the rotational spin of a fermion, like an electron or positron, we get a value, in units of $\hbar/2$, of ± 1. So taking into account both the orientation and the hidden variables, the read-out function for detector A can be written $A(\hat{a}, \lambda) = \pm 1$, and for B we write $B(\hat{b}, \lambda) = \pm 1$. The

[11] The author relied as much on the outline given by Griffiths [8] as on the original paper by Bell for this outline. The logical structure and some of the notation for the proof given is derivative of the version in Griffiths, which the author found to be the clearest.

[12] λ is obtained by isolating away all correlations in the particle system. After we've accounted for all corelations, the unknowns that are left over are by assumption completely noncorrelated, which is why we can call them random. Here we see local realism creeping into the argument. Indeed, the assumption that we can define λ in this way will allow us to separate the measurement functions in our experiments into products. But that assumption is itself predicated on the assumption that each particle enjoys its own separate and well-defined reality. Note that the proof of the first theorem in the Appendix gives away the game before we even get started.

average value of the product of spins, taking into account both the detector orientations and the set of random variables represented by λ, is given in classical probability theory by integrating the product of $A(\hat{a}, \lambda)$ and $B(\hat{b}, \lambda)$ over the probability space, weighted by the probability density $\rho(\lambda)$[13],

$$P(\hat{a}, \hat{b}) = \int d\lambda \rho(\lambda) A(\hat{a}, \lambda) B(\hat{b}, \lambda). \tag{2}$$

If $\hat{a} = \hat{b}$, then $A(\hat{a}, \lambda) = -B(\hat{a}, \lambda)$. Therefore, we can rewrite Equation (2) in terms of A only,

$$P(\hat{a}, \hat{b}) = -\int d\lambda \rho(\lambda) A(\hat{a}, \lambda) A(\hat{b}, \lambda).$$

Now we consider another unit vector $\hat{c}$; we can in the same fashion construct an expression for $P(\hat{a}, \hat{c})$, and then subtract $P(\hat{a}, \hat{c})$ from $P(\hat{a}, \hat{b})$,

$$P(\hat{a}, \hat{b}) - P(\hat{a}, \hat{c}) = -\int d\lambda \rho(\lambda) [A(\hat{a}, \lambda) A(\hat{b}, \lambda) - A(\hat{a}, \lambda) A(\hat{c}, \lambda)]. \tag{3}$$

Because the values of the read-out functions are constrained to be equal to ± 1, the square of any of those functions is equal to 1. In particular, $A(\hat{b}, \lambda)^2 = 1$, and we can factor the integrand in Equation (3),

$$P(\hat{a}, \hat{b}) - P(\hat{a}, \hat{c}) = -\int d\lambda \rho(\lambda) [1 - A(\hat{b}, \lambda) A(\hat{c}, \lambda)] A(\hat{a}, \lambda) A(\hat{b}, \lambda). \tag{4}$$

Since $-1 \leq A(\hat{a}, \lambda) A(\hat{b}, \lambda) \leq 1$, and since $\rho(\lambda)[1 - A(\hat{b}, \lambda) A(\hat{c}, \lambda)] \geq 0$, the absolute value of the integral in Equation (4) is bounded from above by,

$$\int d\lambda \rho(\lambda) [1 - A(\hat{b}, \lambda) A(\hat{c}, \lambda)],$$

and therefore,

$$\left| P(\hat{a}, \hat{b}) - P(\hat{a}, \hat{c}) \right| \leq \int d\lambda \rho(\lambda) [1 - A(\hat{b}, \lambda) A(\hat{c}, \lambda)].$$

Using the fact that $\rho(\lambda)$ is a probability density, we can simplify this. Indeed we now have,

$$\left| P(\hat{a}, \hat{b}) - P(\hat{a}, \hat{c}) \right| \leq 1 + P(\hat{b}, \hat{c}). \tag{5}$$

[13] Other than normalization, we do not know anything about what the probability space or density function look like; however, we will never have to explicitly calculate the integral, so our ignorance is of no consequence.

This is *Bell's inequality*. Griffiths calls the proof "stunningly simple". One of the author's professors, Ciriyam Jayaprakash, recently said something to the effect that the proof can be read and understood by anybody, but a person who can give it the first time apparently only comes around every thirty years. The proof as given in Bell's original paper is not *quite* as accessible, and the form of the result is not as immediately useful as Equation (5) will be to us.

For now we are able to refute the claim that hidden variables can account for the behavior of two spin-1/2 particles in an entangled state. Bell's inequality applies to repeated measurements of a system wherein the *assumed* set of hidden variables defines the probability density function $\rho(\lambda)$. So suppose that quantum mechanics is dependent on some set of hidden variables to be complete. Now we take a large number of readings of the example entangled state using our detectors, with the following stipulation: For half of all the readings we will have the detectors oriented orthogonally, and for the other half we will have the detectors angled at 45 degrees to one another in the plane defined by the orthogonal orientation[14]. Let $\hat{a}$ be the orientation of the fixed detector, and let $\hat{b}$ and $\hat{c}$ be the orientations of the second detector, with $\hat{b}$ orthogonal to $\hat{a}$ and $\hat{c}$ angled at 45 degrees. With this arrangement, we know both from theoretical prediction, (see 6.2), and experimental results that we obtain,

$$P(\hat{a}, \hat{b}) = -\hat{a} \cdot \hat{b} = -Cos(90°) = 0.$$

We also have,

$$P(\hat{a}, \hat{c}) = P(\hat{b}, \hat{c}) = -0.707.$$

Applying equation (5), we then have,

$$0.707 \not\leq 1 - 0.707 = 0.293.$$

Remember that Bell's inequality applies to any system where hidden variables are assumed. We assumed hidden variables, took some measurements, applied Bell's inequality, and now we see that this inequality is violated. Therefore, either quantum mechanics is *wrong* as opposed to being merely incomplete, or quantum mechanics does not rely on hidden variables for its results. The best experimental evidence to date agrees with the

[14]It would be valid to object to any process used to decide when to move the detectors during the measurement session. Analysis of possible bias introduced by the selection process is beyond the scope of our discussion.

prediction of quantum mechanics. This is the ultimate test for a scientific theory. Therefore, we are forced to dispense with hidden variables as an explanation for the behavior of the entangled state system.

5 Conclusion

As anyone can attest who has argued against the existence of Bigfoot, free markets, or the Chupacabra, and lost the argument, there is no way to prove a negative. Even though nonlocality is a fact, it is also a mystery. Humans, and especially rational humans, desire cause and effect. We want there to be a nameable and quantifiable mechanism that will explain how the wave function collapses instantly. Therefore rational humans will continue to conjure such mechanisms. Bell himself mentions four positions that a person might take on "this business" of nonlocality, and he himself expresses a preference for the position that quantum mechanics might be wrong in certain critical, and as yet untested, situations[15]. However, he also points out that the experimental evidence for such a position is not encouraging. The paper in which he lays out the four positions was written just as Alain Aspect was concluding an experiment in which he was able to change the detector settings while the second entangled particle, a photon, was in flight, and after the first particle had been measured. The results of that experiment confirmed that measurements of an entangled system violate Bell's inequality[16].

6 Appendix

Griffiths [8] gives several problems as exercises that help demonstrate important properties of the singlet state. The author has given solutions to these exercises below.

6.1 The singlet spin equation is not separable

The singlet spin state,

$$|\psi\rangle = \frac{1}{\sqrt{2}}\{|\uparrow_-\downarrow_+\rangle - |\downarrow_-\uparrow_+\rangle\}, \tag{6}$$

[15] Paper 16 in [7].

[16] Aspect wrote an excellent introduction to the edition of [7] that the author referenced for this paper.

is not "separable" into a disentangled product of two one-particle states. We prove the following:

Consider a two-level system, $|\phi_a\rangle$ and $|\phi_b\rangle$, with $\langle\phi_i|\phi_j\rangle = \delta_{ij}$. The two-particle state,

$$\alpha\,|\phi_a(1)\rangle\,|\phi_b(2)\rangle + \beta\,|\phi_b(1)\rangle\,|\phi_a(2)\rangle\,,$$

(with $\alpha \neq 0$ and $\beta \neq 0$), *cannot* be expressed as a product,

$$|\psi_r(1)\rangle\,|\psi_s(2)\rangle\,,$$

for *any* one-particle states $|\psi_r\rangle$ and $|\psi_s\rangle$.

Proof. Write $|\psi_r(1)\rangle$ and $|\psi_s(2)\rangle$ as linear combinations of $|\phi_a\rangle$ and $|\phi_b\rangle$,

$$\begin{aligned}|\psi_r(1)\rangle &= \kappa\,|\phi_a(1)\rangle + \lambda\,|\phi_b(1)\rangle\,,\\ |\psi_s(2)\rangle &= \mu\,|\phi_a(2)\rangle + \nu\,|\phi_b(2)\rangle\,.\end{aligned}$$

Then we have,

$$\begin{aligned}|\psi_r(1)\rangle\,|\psi_s(2)\rangle = \kappa\mu\,|\phi_a(1)\rangle\,|\phi_a(2)\rangle + \kappa\nu\,|\phi_a(1)\rangle\,|\phi_b(2)\rangle\\ + \lambda\mu\,|\phi_b(1)\rangle\,|\phi_a(2)\rangle + \lambda\nu\,|\phi_b(1)\rangle\,|\phi_b(2)\rangle\,.\end{aligned}$$

To obtain the two-particle singlet state from this expression, we need the first and fourth terms to vanish, and the only way for that to happen is for one from each set $\{\kappa, \mu\}$ and $\{\lambda, \nu\}$ to be equal to zero. But we assumed that $\alpha = \kappa\nu \neq 0$ and $\beta = \lambda\mu \neq 0$, so we have a contradiction. Therefore the claim is shown. □

6.2 The average value of the product of spins

We want to be able to calculate the average value of the product of "spin readings" for detectors which have an arbitrary relative orientation. We prove the following:

Consider again Equation (6). Let $S_a^{(1)}$ be the component of the spin of particle 1 in the direction defined by the unit vector $\hat{a}$. Let $S_b^{(2)}$ be the component of the spin of particle 2 in the direction defined by the unit vector $\hat{b}$. Then,

$$\langle S_a^{(1)} S_b^{(2)}\rangle = -\frac{\hbar^2}{4} cos\theta,$$

where θ is the angle between $\hat{a}$ and $\hat{b}$.

Proof. Since we are looking for a value dependent on the relative angle θ between the two spin operators, we can choose one of those operators, $\hat{a}$, to be in the $\hat{z}$ direction. This greatly simplifies the algebra. It is also helpful to rewrite the singlet state, since the arrow notation will just get in the way. We write,

$$|\psi\rangle = \frac{1}{\sqrt{2}} \left\{ \left| \begin{array}{c} 1 \\ 0 \end{array} \right\rangle_- \left| \begin{array}{c} 0 \\ 1 \end{array} \right\rangle_+ - \left| \begin{array}{c} 0 \\ 1 \end{array} \right\rangle_- \left| \begin{array}{c} 1 \\ 0 \end{array} \right\rangle_+ \right\}.$$

With our choice of coordinates, we have our two spin operators expressed as,

$$S_a^{(1)} = \left(\begin{array}{cc} 1 & 0 \\ 0 & \text{-}1 \end{array} \right), \quad S_b^{(2)} = \left(\begin{array}{cc} 0 & e^{\text{-}i\theta} \\ e^{i\theta} & 0 \end{array} \right).$$

We want to evaluate $\langle\psi| S_a^{(1)} S_b^{(2)} |\psi\rangle$, and we will do it in two steps so it is clear how we get the scalar value at the end. Applying the two spin operators to $|\psi\rangle$, we get,

$$S_a^{(1)} S_b^{(2)} |\psi\rangle = \frac{\hbar^2}{4\sqrt{2}} \left\{ \left| \begin{array}{c} 1 \\ 0 \end{array} \right\rangle_- \left| \begin{array}{c} e^{i\theta} \\ 0 \end{array} \right\rangle_+ - \left| \begin{array}{c} 0 \\ \text{-}1 \end{array} \right\rangle_- \left| \begin{array}{c} 0 \\ e^{\text{-}i\theta} \end{array} \right\rangle_+ \right\}.$$

Now we take the inner product of $\langle\psi|$ and the above expression,

$$\frac{1}{\sqrt{2}} \left\{ \left\langle \begin{array}{c} 0 \\ 1 \end{array} \right|_+ \left\langle \begin{array}{c} 1 \\ 0 \end{array} \right|_- - \left\langle \begin{array}{c} 1 \\ 0 \end{array} \right|_+ \left\langle \begin{array}{c} 0 \\ 1 \end{array} \right|_- \right\} \frac{\hbar^2}{4\sqrt{2}} \left\{ \left| \begin{array}{c} 1 \\ 0 \end{array} \right\rangle_- \left| \begin{array}{c} e^{i\theta} \\ 0 \end{array} \right\rangle_+ - \left| \begin{array}{c} 0 \\ \text{-}1 \end{array} \right\rangle_- \left| \begin{array}{c} 0 \\ e^{\text{-}i\theta} \end{array} \right\rangle_+ \right\},$$

which reduces to,

$$-\frac{\hbar^2}{8}(e^{i\theta} + e^{-i\theta}) = -\frac{\hbar^2}{4} cos\theta.$$

□

References

[1] Letter from Einstein to Max Born, 3 March 1947, **The Born-Einstein Letters**, Macmillian, London (1971).

[2] Many contributors, *Principal of Locality*, available at http://en.wikipedia.org/wiki/Principle_of_locality.

[3] Einstein, E., Podolsky, B., and Rosen, N., *Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?*, Physical Review, 47:777-780 (1935).

[4] J. S. Bell, *On the Einstein Podolsky Rosen Paradox*, Physics **1**, 195 (1964).

[5] Stephen Jablonsky, *complementarity principle*, available at http://www.britannica.com/nobel/micro/138_65.html.

[6] Fine, Arthur, *The Einstein-Podolsky-Rosen Argument in Quantum Theory*, The Stanford Encyclopedia of Philosophy (Summer 2004 Edition), Edward N. Zalta (ed.), available at http://plato.stanford.edu/archives/sum2004/entries/qt-epr/.

[7] J.S. Bell, **Speakable and Unspeakable in Quantum Mechanics : Collected Papers on Quantum Philosophy** , Cambridge University Press, Cambridge (2004).

[8] David J. Griffiths, **Introduction to Quantum Mechanics**, Prentice Hall, New York (1994).