Physics 827 Problem Set 7 Due Monday 11/30/2009

8.1 Shankar 10.2.1 (page 258) and 10.2.3 (page 260) (5+3 points) **In 10.2.3 (2) just re-express the first three states in terms of spherical polar coordinates. The degeneracy question needs a simple combinatoric trick; think about it but do not submit.**

8.2 This problem deals with the one-dimensional simple harmonic oscillator. Write down the Heisenberg equations of motion for the operators $a_H(t)$ and $a_H^\dagger(t)$. (See page 490 of the text to recall the equation of motion.) Solve them and determine the time dependence of $a_H(t)$ and $a_H^\dagger(t)$. Either from these or by solving the equation of motion directly. find the time dependence of $X_H(t)$ and $P_H(t)$ the position and momentum operators in the Heisenberg representation. Make a precise connection to the classical results.

Find $\langle X_H(t) X_H(0)\rangle$ in the ground state.

Define the time-ordered product as follows:
$T[X_H(t_1)X_H(t_2)] = X_H(t_1)\,X_H(t_2)$ if $t_1 > t_2$ and equal to $X_H(t_2)\,X_H(t_1)$ if $t_2 > t_1$. At the moment it is just a definition but in field theory this will become the Feynman propagator. Find the expectation value of this propagator in the ground state. (8 points)

Please do not submit: If you want to be a theorist do the evaluations of the products of position operators at finite temperatures. Work out the expectation value (in the ground state) of the product of four position operators $X(t_4)X(t_3)X(t_2)X(t_1)$ with $t_4 > t_3 > t_2 > t_1$; you will have checked the baby case of Wick's theorem. Find the equation of motion obeyed by $T[X_H(t_1)X_H(t_2)]$.

8.3) **Bogoliubov Transformation:**
You are given that $[a,\, a^\dagger] = 1$.
(a) Let $b = ua + va^\dagger$ where u and v are real. What is $b^\dagger$? Find the condition on u and v such that b and $b^\dagger$ obey the same commutation relation as a and $a^\dagger$. (One says that b and a are canonical Bose operators. The transformation that preserves the commutation relation is said to be a canonical transformation.)

(b) Consider the Hamiltonian

$$H_1 = \hbar\omega\left(a^\dagger a + \frac{1}{2}\right) + \Delta\left(aa + a^\dagger a^\dagger\right).$$

Show that ω and Δ are real. Use the transformation defined in (a) to diagonalize the Hamiltonian, i.e., write it in the form

$$H_1 = \hbar\Omega\left(b^\dagger b + \frac{1}{2}\right)$$

by a suitable choice of u and v. Find u, v, and Ω in terms of the given parameters ω and Δ. (8 points)

8.4) Shankar 12.2.3 Please review an undergraduate text chapter on angular momentum. Do just the first part of the problem. (2 points)