6.1) Consider the two delta-function potential,

$$V(x) = va\delta(x+a) + va\delta(x-a)$$

where $v < 0$.
(a) Find the transcendental equation that determines the ground state (Use the parity of the ground state wave function to choose the form of the wave function in the region $(-a,\, a)$ intelligently.) Is there always a bound state?
(b) Repeat part (a) for the first excited state or odd parity states in general.
(c) When $|v| >> \hbar^2/(ma^2)$ find the even and odd parity energies and the splitting between them. Compare with the discussion for the double well in class. (10 points)

6.2) For the infinite potential well compute and plot the probability of finding a momentum value between p_0 and $p_0 + dp$ for the ground state and the first excited state. Please use Mathematica or Tables to do the integrals. (6 points)

6.3) This is easy and done in class. Please do it yourselves: Exercises 5.3.3. and 5.3.4 on page 167. (4 points)

6.4) Consider the potential step discussed in Shankar (pages 168-169) and find the reflection and transmission coefficients. Define them carefully starting from the definitions in terms of the currents.

Find the reflection coefficient and transmission coefficient and check that the sum is unity. Explain precisely why the sum should be unity. *One extra point for a really interesting observation about the results for T and R.*

Now consider an incident particle from the right and calculate the reflection and transmission coefficients. Compare your answer with the answer for a particle incident from the left. (6 points)

This part is harder version of the question raised by John Campbell: Consider a smooth local potential $V(x)$ that is non-vanishing between $[-a/2,\, a/2]$ and zero outside but otherwise arbitrary. A plane wave incident from the left has a transmission amplitude t and a reflection amplitude r. [The transmitted wave has the form te^{ikx} and the reflected wave is re^{-ikx} when the incident wave is e^{ikx}. The complex coefficients are referred to as the amplitudes] If a plane wave is incident only from the right the transmission coefficient is given to be τ and the reflection coefficient is ρ. What general relations can you derive between $\{\rho, \tau\}$ and $\{r, t\}$? You cannot solve for r, t, etc. since you are not given $V(x)$. You have to use general properties of the Schrödinegr equation. (6 points)

6.5) In this problem you will compare the probability for tunneling through a rectangular barrier of height V_0 and width $2a$ with that for a parabolic barrier $V_0 \frac{a^2-x^2}{a^2}$ between $-a$ and a and zero outside. It is not pleasant solving the parabolic barrier analytically. So use the crude WKB approximation (discussed in Monday's lecture) to compare the relative transmission coefficients at an energy $E < V_0$. Pick some values and get some result that you can express simply. (8 points)

Recall that the transmission coefficient can be approximated by

$$T \approx e^{-2\int_{x_1}^{x_2} dx \sqrt{\frac{2m}{\hbar^2}(V(x)-E)}}$$

where x_1 and x_2 are the classical turning points.