Physics 827 Problem Set 5 Due Friday 11/06/2009

The first two problems almost made it to the midterm

5.1 Consider the state $e^{ip_0X/\hbar}\,|p\rangle$ where $|p\rangle$ is an eigenstate of the momentum operator P, X is the position operator and p_0 is real. Show that this is also an eigenstate of P and find the eigenvalue. (3 points)

5.2 Consider the diffusion equation for the density $n(x,t)$ in one dimension

$$\frac{\partial n}{\partial t} = D\,\frac{\partial^2 n}{\partial x^2}$$

where D is the diffusion constant. You are given $n(x,0) = n_0\,a\,\delta(x)$. (Why is it convenient to write it in this form? What is the dimension of a? Shankar does something similar in Exercise 5.2.3) Find $n(x,t)$; in particular determine how the width of the density profile evolves in time. (8 points)

5.3 Find the transcendental equation that determines the energy eigenvalues of the odd bound states for a potential well of depth $-V_0$ and width a symmetrically disposed about the origin considered in the lecture. You are urged to do this problem starting from the form of the wave functions in the different regions without looking at your notes or any book. Consider an electron in a well of width $a = 1nm$. What is the minimum well depth at which the first odd bound state appears? (5 points)

5.4 Exercise 5.2.3 on page 163. (6 points)
If you did not do the problem on the midterm fully please find the momentum distribution function and its width. (2 points)

5.5 Given $\psi(x) = R(x)\,e^{i\phi(x}$ where $R(x)$ and $\phi(x)$ are real-valued functions find the (probability) current density. The result is worth remembering. Generalize it to a three-dimensional wave function. We have suppressed the time dependence for convenience. (4 points)