Physics 827 Problem Set 4 Due Friday 10/23/2009

4.1. A system is described by the Hamiltonian H and we consider an observable denoted by S_y. The two operators are given by

$$H = \begin{pmatrix} 0 & \epsilon \\ \epsilon & 0 \end{pmatrix} \quad \text{and} \quad S_y = \frac{\hbar}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}.$$

(a) What is the dimensionality of the Hilbert space to which a state vector describing the system belongs? Consider the state of the system described by $|\psi_1\rangle = \frac{1}{\sqrt{5}}\begin{pmatrix} 2 \\ i \end{pmatrix}$. If one measures the observable denoted by S_y what are the possible results of the measurement and what are the corresponding probabilities? Explain precisely what it means to calculate probabilities.

(b) S_y is measured on the ensemble described in (a) at $t = 0$. Those copies of the ensemble that yield the larger of the eigenvalues is selected. What is the state of the system immediately after the measurement? Denote this state by $|\psi(t = 0)\rangle$.

(c) What is $|\psi(t)\rangle$ for $t > 0$? Explain clearly the logic of your calculation.

(d) At time t you measure S_y for an ensemble described by $|\psi(t)\rangle$ from part (c). Calculate the probability of measuring the the two eigenvalues and sketch them. Is there any connection between these results and the probabilities you found in (a)? If not why not?

(e) Find an explicit expression for the time evolution operator for this system as a 2×2 matrix. Use this to determine $|\psi_1(t)\rangle$ where $|\psi_1(t = 0)\rangle = |\psi_1\rangle$ given in part (a).

(f) Do you think an experimentalist can actually prepare a spin system in the state given in (a). If yes describe how one might do it.

(4.2) and (4.3) Exercises 4.2.2 and 4.2.3 on page 139.

(4.4)Exercise 5.2.1 (tentative)