PHYSICS 827 HW1
Due Friday 10/02 by 4:59PM in Mr. Nick Harmon's mailbox

1.1 Shankar Exercise 1.3.4 (p17) (10 points)

1.2 Shankar Exercise 1.6.2 (p27) (5 points)

1.3 Shankar Exercise 1.6.4 (p29) (5 points)

1.4 (10 points) Consider a three-dimensional Hilbert space spanned by the orthonormal basis $|1\rangle$, $|2\rangle$, and $|3\rangle$. The kets $|\psi\rangle$ and $|\phi\rangle$ are given by

$$|\psi\rangle = a|1\rangle + b|2\rangle + a|3\rangle \quad \text{and} \quad |\phi\rangle = b|1\rangle - a|2\rangle$$

where a and b are complex constants.

(a) Determine $\langle\psi|$, $\langle\phi|$, $\langle\psi|\phi\rangle$ and $\langle\phi|\psi\rangle$. Under what conditions are $|\psi\rangle$ and $|\phi\rangle$ orthogonal?

(b) Express $|\psi\rangle\langle\phi|$ as a 3×3 matrix, i.e., determine all the matrix elements in the given orthonormal basis.

(c) Consider the operator $P \equiv |\psi\rangle\langle\psi| + |\phi\rangle\langle\phi|$. Is P Hermitian? What is P^2? Give a simple argument (no explicit calculations) to show that P has a zero eigenvalue.

1.5 (6 points) Consider the operator $U = e^{iaH}$ where a is a real number and H is a Hermitian operator. The exponential is defined by the power series. Show (carefully) that U is unitary for finite-dimensional vector spaces. This (and the generalization to Hilbert spaces) is a very important result for quantum mechanics.

1.6. (4 points) Consider the Pauli spin matrices given by

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \text{and} \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad . \tag{1}$$

Find σ_x^2. use this result to determine $e^{i\alpha\sigma_x}$ as a linear combination of σ_x and the identity matrix for real α.

Optional: Check that $\sigma_x \sigma_y = i\sigma_z$. What is $\sigma_y \sigma_x$? Assume the corresponding results for the cyclic permutations. Let $\hat{n} = (n_x, n_y, n_z)$ be a unit vector in three dimensions. What is $(\vec{\sigma} \cdot \hat{n})^2$? Use this result to determine $e^{i\alpha\vec{\sigma}\cdot\hat{n}}$ for real α.