5.1) Related to Griffiths Problem 2.33. Consider the barrier problem and the tunneling case with \( E < V_0 \). Derive the given result as follows: Assume that you have derived Equation 2.169 for the potential well case. By changing the sign of \( V_0 \) (from well to barrier) and being clear about the sign of \( E - V_0 \) show that the transmission coefficient for the tunneling problem can be obtained.

Hint: What is \( \sin(ia) \) for real \( a \)?

(b) Show that for \( \kappa a \) large where \( \frac{\hbar^2 a^2}{2m} = V_0 - E \) the transmission coefficient in the tunneling case (given in the book) can be written approximately as \( T_0 e^{-4\kappa a} \) and determine \( A_0 \) in terms of \( E_0 \) and \( V_0 \). Explain clearly the argument of the exponential dependence.

(c) For an electron the transmission coefficient (probability to tunnel) is given to be 0.10. Suppose we use a proton (changing voltages so that it encounters a barrier) with the same energy and the same \( V_0 \). Estimate the transmission coefficient. (Use the approximate exponential form.)

5.2) Please read the notes on the potential step (particle incident from the left, the standard case in Problem 2.34) and answer the following:

(a) Define the reflection amplitude \( r \) to be \( r \equiv B/A \). Note that the reflection coefficient \( R = |r|^2 \). Find \( r \) explicitly and show that it is given by \( (k - i\kappa)/(k + i\kappa) \) for \( E < V_0 \). The form for \( E > V_0 \) is given in the notes. Determine the magnitude and phase of \( r \) for \( E < V_0 \) and \( E > V_0 \) separately. The phase is interpreted as the change in phase due to reflection.

(b) Plot the phase as a function of \( E/V_0 \) for all \( E \). What happens to the phase change when \( V_0 \to \infty \)? Try to compare this result for the infinite step case with a classical wave on a string incident on a hard wall.

(c) Extra credit problem from the exam: Consider a wave function for the incident part given by \( A_1 e^{ik_1 x} \). Assume incidence from the left on a potential step of height \( V_0 \) with energy \( E_1 = \frac{k^2_1 a^2}{2m} > V_0 \). If the transmitted wave has the form \( C_1 e^{i q_1 x} \) then you know \( C_1/A_1 \) from the notes. Suppose the incident wave is a superposition (not two particles!) \( A_1 e^{i k_1 x} + A_2 e^{i k_2 x} \) with \( E_2 = \frac{k^2_2 a^2}{2m} > V_0 \). Determine the transmitted wave. Hint: The Schrödinger equation is linear.

(5.3) Griffiths 2.47 part (a) only.