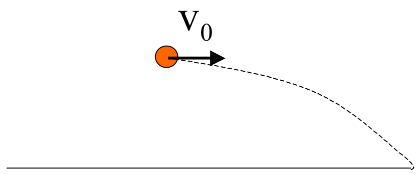
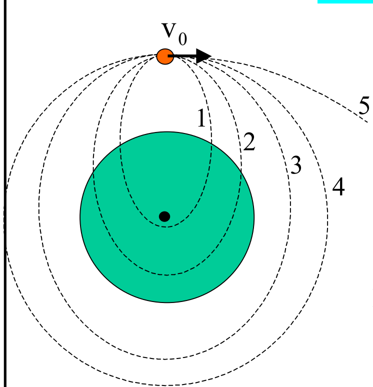
Projectile Motion



Projectile Motion:

- We assumed that the projectile was fired over a horizontal surface
- This is approximately true for a projectile fired over the surface of the earth, for projectiles with small initial horizontal velocities

Satellite Motion



If the projectile is fired far from the earth surface (so it won't hit buildings, mountains, etc), and fired with large enough initial velocity, the curvature of the earth does matter

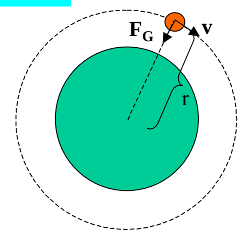
For paths 1 and 2, the projectile will land on the earth.

For paths 3 and 4, the projectile has a large enough initial horizontal velocity, that the projectile continues around the earth without ever landing. The projectile has become a satellite.

For path 5, the initial velocity is large enough that the earth's gravity cannot pull the projectile back.

Paths 1,2,3,4 are ellipses. Path 4 is a special case of an ellipse - a circle. We will study this in detail next.

Satellites: Uniform Circular Motion plus Gravitation



What keeps a satellite up? Its high speed.

What keeps at satellite from flying away? Gravitation.

For a satellite of mass m, circling a planet of mass m_P at a radius r:

$$\sum F = ma$$

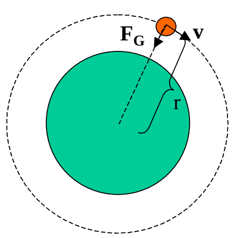
$$\frac{Gmm_P}{r^2} = \frac{mv^2}{r}$$

$$\frac{G\cancel{m}m_P}{r^2} = \frac{\cancel{m}v^2}{r}$$

$$v = \sqrt{\frac{Gm_P}{r}}$$

(orbital speed)

Satellites: Uniform Circular Motion plus Gravitation



We can also relate the orbital speed to the time for 1 revolution:

$$v = \frac{\text{distance in 1 rev}}{\text{time for 1 rev}}$$

$$v = \frac{(2\pi r)}{T}$$

$$v = \sqrt{\frac{Gm_P}{r}} = \frac{2\pi r}{T}$$

$$T = 2\pi r\sqrt{\frac{r}{Gm_P}}$$

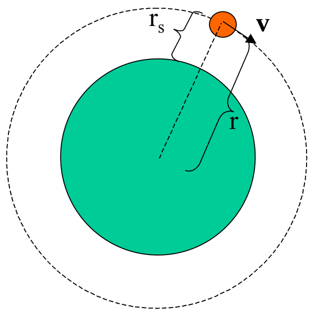
$$T = \sqrt{\frac{4\pi^2 r^3}{Gm_P}}$$

Larger orbits have:
1) Slower speeds.
2) Longer times to complete one revolution.

Example: Putting a satellite into a circular orbit.

We want to put a 5000kg communications satellite into a circular orbit, 300km above the earth's surface.
The mass of the earth is 5.98 x 10^{24} kg and its radius is 6380 km.

A) What speed must it have?
B) What is its period of motion?
C) What is the radial acceleration of the satellite in this orbit?



The radius of the satellite's orbit is:

$$r = r_E + r_s = 6380\text{km} + 300\text{km} = 6.980 \times 10^6\,\text{m}$$

Example Continued : Putting a satellite into a circular orbit.

A) The speed of the satellite must be:

$$v = \sqrt{\frac{Gm_P}{r}} = \sqrt{\frac{(6.67 \times 10^{-11})(5.98 \times 10^{24})}{6.98 \times 10^6}} = 7730\text{m/s}$$

B) The period of motion is:

$$T = \sqrt{\frac{4\pi^2 r^3}{Gm_P}} = \frac{2\pi\, r}{v} = \frac{2\pi\,(6.98 \times 10^6)}{7730} = 5430\ \text{s} = 90.5\ \text{min}$$

C) The radial acceleration is (since this is uniform circular motion):

$$a = \frac{v^2}{r} = \frac{(7730)^2}{6.98 \times 10^6} = 8.94\text{m/s}^2$$

Why is this not equal to 9.80m/s² ?