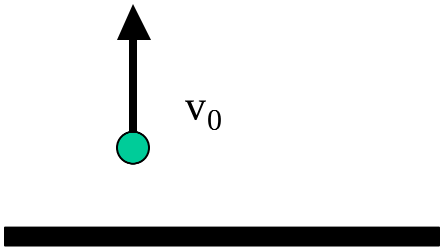
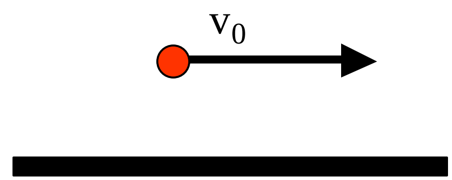
Motion in a vertical plane

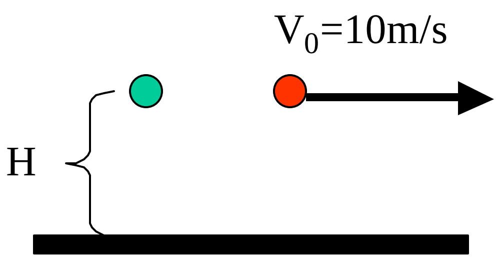
We've done this:



Now let's do this:



Question: Two objects, both at the same height H. One is dropped from rest. Other is thrown horizontally with V_0=10m/s. Which hits the ground first?



<u>Both hit the ground at the same time.</u> Why?

- Both start out with the same vertical velocity (=0)
- Both have the same vertical acceleration (=9.80/ms^2)
- Therefore both take the same amount of time to fall the vertical distance H
- Of course, the red object travels some distance in x, while the green object does not travel in x.

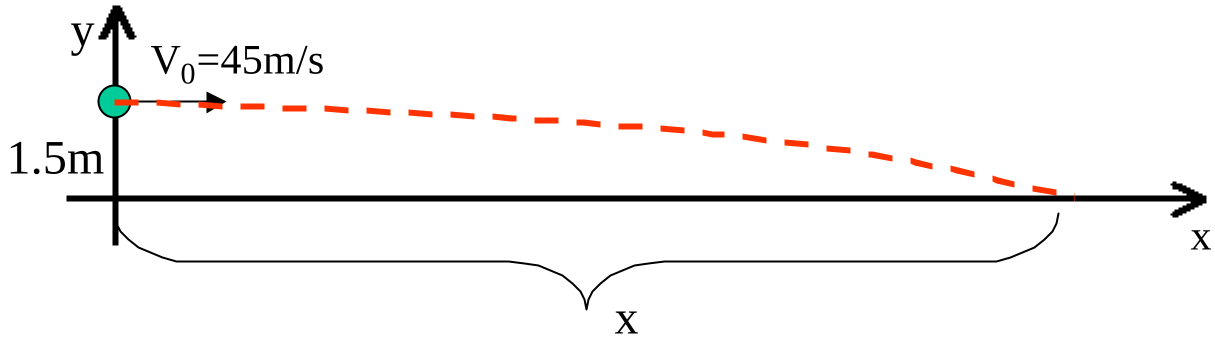
Motion in a vertical plane: Projectile Motion

Projectile Motion:

1) Describes the motion of an object near the earth's surface where the acceleration due to the earth's gravity is approximately constant.
2) It is motion in two dimensions:
   - **vertical:** In this dimension there is ***a downward acceleration due to gravity.*** This means that the vertical component of the velocity will change with time. If initially directed upward, it will decrease, becoming zero at the maximum height, and reversing direction and then increasing in magnitude.
   - **horizontal:** In this dimension there is ***no acceleration*** This means the horizontal component of the velocity remains constant throughout the entire path of the object.
3) The path of the object is called **the trajectory**, and for projectile motion this path is **parabolic.**
4) The horizontal motion and vertical motion of the object are *completely independent*. The only connection between the two is that they occur at the same time. For example, if you find how long it takes an object to rise to its maximum height, you can use this time to determine how far the object has traveled horizontally.

Example: A baseball is thrown with a horizontal velocity of 45m/s, from a height of 1.5m.
A) How long till the ball hits the ground?
B) How far does it travel horizontally before hitting the ground?

I) Draw a picture:



II) Write what you know:

$$x_0 = 0$$
$$x = ?$$
$$v_{x0} = 45\text{m/s}$$
$$v_x = 45\text{m/s}$$
$$a_x = 0$$
$$t = ?$$

$$y_0 = 1.5\text{m}$$
$$y = 0$$
$$v_{y0} = 0$$
$$v_y = ?$$
$$a_y = -9.80\text{m/s}^2$$
$$t = ?$$

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Example (continued):

First find how long it will take for the ball to fall 1.5m in the y-direction:

$$y = y_0 + v_{y0}t + \frac{1}{2}at^2$$

$$1.5 = 0 + 0 + \frac{1}{2}(-9.80)t^2$$

$$t = \sqrt{\frac{2(1.5)}{9.8}} = 0.55s$$

NOTE: You will get this time no matter how fast the initial x-velocity is.

Now you can find how far it travels in the x-direction:

$$x = x_0 + v_{x0}t + \frac{1}{2}a_x t^2$$

$$\text{but } a_x = 0, \text{ so}$$

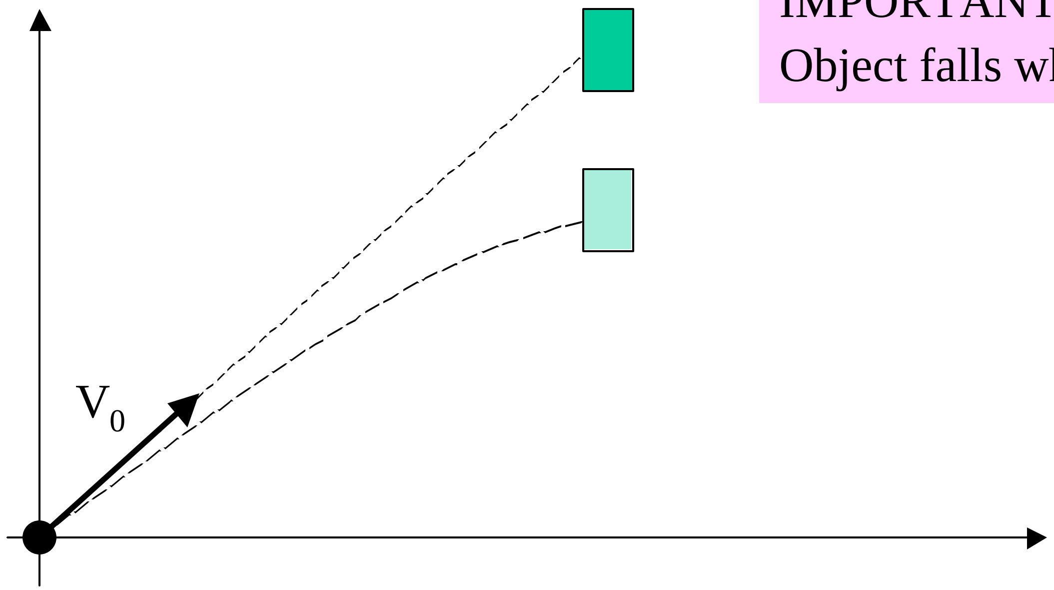
$$x = x_0 + v_{x0}t$$

$$x = 0 + (45)(0.55)$$

$$x = 25m$$

DEMO:

IMPORTANT:
Object falls when projectile is fired.



Projectile is aimed **directly** at hanging object

Equation describing y-motion of projectile:

$$y = y_0 + v_{y0}t + \frac{1}{2}at^2$$

$$y - y_0 = v_{y0}t - \frac{1}{2}gt^2$$

Straight line path

Deviation from Straight line path

Equation describing falling object:

$$y = y_0 + v_{y0}t + \frac{1}{2}at^2$$

$$y - y_0 = -\frac{1}{2}gt^2$$

Same as:

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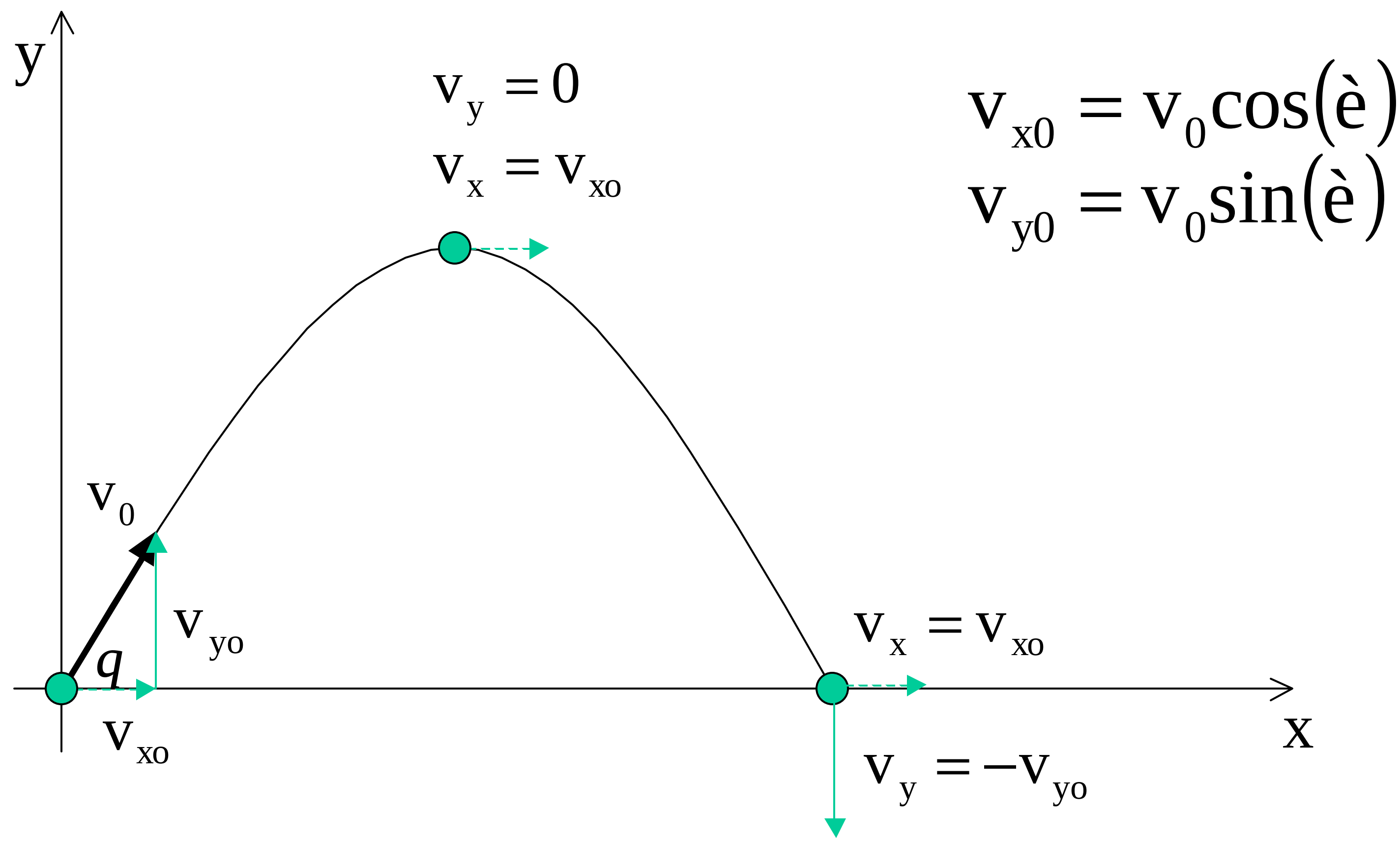
Demo Explanation

- The object falls a distance in time t
- Which is the same distance as the deviation of the projectile from its straight line path.
- So by the time the projectile gets to the point in x where the falling object is
- It is at the same point in y as the falling object .
- Got it?

"General" Projectile Motion

Object is thrown with an initial speed V_0 at an angle θ to the horizontal. What is its range (the horizontal distance traveled)?



Write what you know in x...

$$x_0 = 0$$
$$x = ?$$
$$v_{x0} = v_0 \cos(q)$$
$$v_x = v_{x0}$$
$$a_x = 0$$
$$t = ?$$

…and in y-direction

$$y_0 = 0$$
$$y = 0$$
$$v_{y0} = v_0 \sin(q)$$
$$v_y = ? \text{(known in this case)}$$
$$a_y = -9.80 \text{m/s}^2$$
$$t = ?$$

Equations for x-motion

$$x = x_0 + v_{x0} t$$
$$v_x = v_{x0}$$

Equations for y-motion

$$y = y_0 + v_{y0} t + \frac{1}{2} a t^2$$
$$v_y = v_{y0} + at$$
$$v_y^2 = v_{y0}^2 + 2a(y - y_0)$$

"General" Projectile Motion (continued)

Start with the x-motion:

$$x = x_0 + v_{x0}t$$

$$R = v_0\cos(q\ )t$$

But how do we get the time t?
Look at the y-motion:

$$y = y_0 + v_{y0}t + \frac{1}{2}at^2$$

$$0 = 0 + v_0\sin(è)t - \frac{1}{2}gt^2$$

solve for t

$$t = \frac{2v_0\sin(è)}{g}$$

Now substitute into x-motion equation:

$$R = v_0\cos(è)t$$

$$R = v_0\cos(è)\left[\frac{2v_0\sin(è)}{g}\right]$$

but $2\sin(è)\cos(è) = \sin(2è)$

$$R = \frac{v_0^2\sin(2è)}{g}$$

"General" Projectile Motion (continued)

Result for range: $$R = \frac{v_0^2 \sin(2è)}{g}$$

This equation says that the range is dependent on:

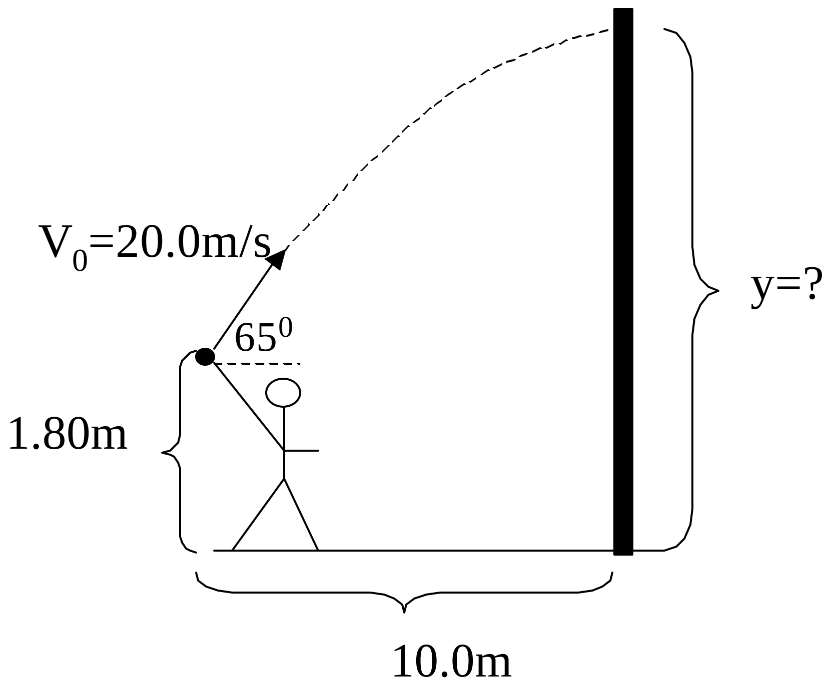
- Angle thrown at
- Speed
- Acceleration due to gravity (on moon things will go farther!)

Also!

- Range is a maximum at $\theta=45^0$ $(\sin(90^0)=1.0)$
- Range at $\theta=30^0$ equals range at $\theta = 60^0$
- Range at general $\theta=\alpha$ equals range at $\theta = 90^0-\alpha$ (why?)

**Do the math: Try q = 10^0 and q = 80^0 ,
and other pairs of angles.**

Example: A ball is thrown with an initial speed of 20.0m/s at an angle of 65^0 above the horizontal. The ball leaves the thrower's hand at a height of 1.80m. At what height will it strike a wall 10.0m away?



$$x_0 = 0$$
$$x = 10.0$$
$$v_{x0} = 20.0\cos(65) = 8.45\text{m/s}$$
$$v_x = 8.45\text{m/s}$$
$$a_x = 0$$
$$t = ?$$

$$y_0 = 1.8$$
$$y = ?$$
$$v_{y0} = 20\sin(65) = 18.1\text{m/s}$$
$$v_y = ?$$
$$a_y = -9.80\text{m/s}^2$$
$$t = ?$$

Example: Continued

Find out how long it takes for the ball to travel 10.0m in the horizontal direction:

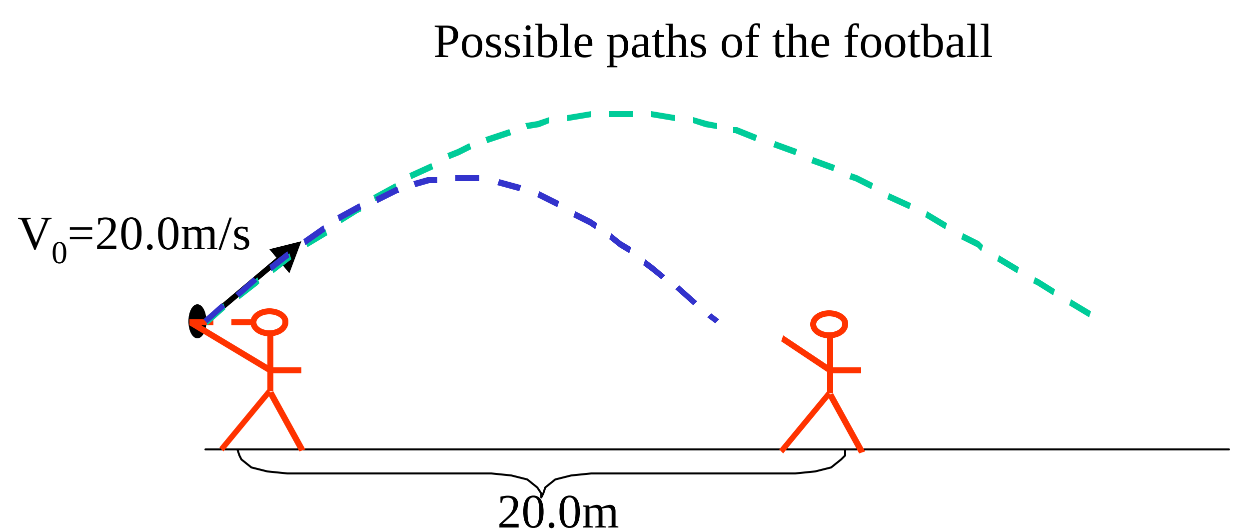
$$x = x_0 + v_{x0} t$$
$$10.0 = 0 + 8.45t$$
$$t = \frac{10.0}{8.45} = 1.18s$$

Then use this time to determine the vertical position of the ball:

$$y = y_0 + v_{y0} t + \frac{1}{2} a t^2$$
$$y = 1.8 + (18.1)(1.18) - \frac{1}{2}(-9.8)(1.18)^2$$
$$y = 16.4\text{m}$$

Example: A football is thrown toward a receiver. The football is thrown with an initial speed of 20.0m/s at an angle of 30^0 above the horizontal. At that instant, the receiver is 20.0m from the quarterback.

In what direction, and with what constant speed shown the receiver run to catch the ball (at the level thrown)?



To answer the question regarding the receiver, we first need to know some things about the football:

1) How far does the football travel horizontally.
2) How long is the football in the air.

Example: Continued

Describe the motion of the football:

$$
\begin{aligned}
x_0 &= 0 \\
x &= ? \\
v_{x0} &= 20\cos 30 = 17.3\text{m/s} \\
v_x &= 17.3\text{m/s} \\
a_x &= 0 \\
t &= ?
\end{aligned}
$$

Define the initial y position of the football as y=0m.

$$
\begin{aligned}
y_0 &= 0 \\
y &= 0 \\
v_{y0} &= 20\sin 30 = 10.0\text{m/s} \\
v_y &= -10.0\text{m/s} \\
a_y &= -9.80\text{m/s}^2 \\
t &= ?
\end{aligned}
$$

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Example (continued)

Find how long in time it takes for the football to go from y=0 (where the quarterback throws it) back to y=0 (where the receiver can catch it).

$$y = y_0 + v_{y0}t + \frac{1}{2}at^2$$

$$0 = 0 + 10.0t - \frac{1}{2}(-9.8)t^2$$

$$t = 2.04\text{s}$$

Use the above time to find out where horizontally the ball is:

$$x = x_0 + v_{x0}t$$

$$x = 0 + (17.3)(2.04)$$

$$x = 35.3\text{m}$$

To catch the ball, the receiver needs to run from 20.0m to 35.3m in 2.04s:

$$v_R = \frac{\text{Ä}x}{\text{Ä}t} = \frac{(35.3\text{m} - 20.0\text{m})}{2.04\text{s}} = +7.5\text{m/s}$$

Since 35.3m > 20.0m, the receiver needs to run away from the quarterback to catch the ball.