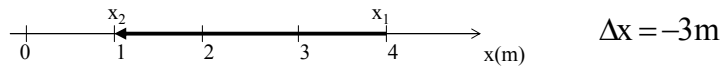
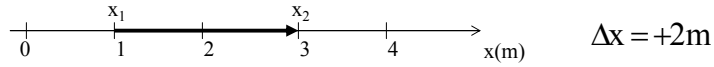


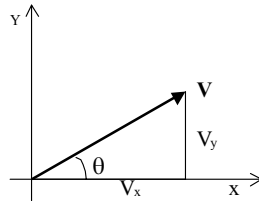
Vectors

Vector: A quantity that has both **magnitude** and **direction**.

- Displacement, velocity, and acceleration are examples of vectors
- In 1-dimension, the direction is specified by a +/- sign



- In 2-dimensions, the direction is specified by an angle relative to one axis (usually the x-axis)



Lecture 4: Vectors

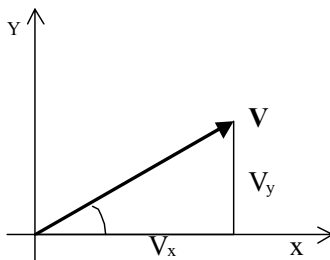
1

A vector is drawn using an Arrow.

- The angle relative to the x-axis gives the direction
- The length of the arrow gives the magnitude of the vector

Notation:

- In the book, the vector \mathbf{V} is represented using boldface: \mathbf{V}
- In class, and in notes, it is easier to represent a vector by writing a symbol with an arrow over it. So the vector \mathbf{V} is represented as: \vec{V}



Lecture 4: Vectors

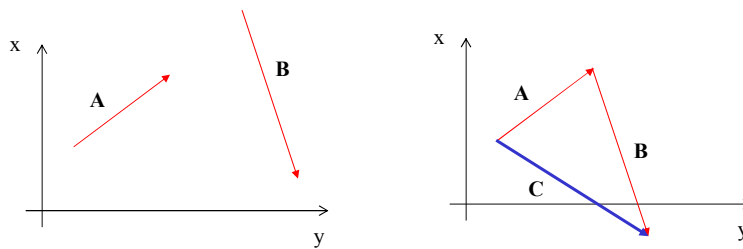
2

Adding Vectors Graphically

Say we wanted to find the sum of two vectors **A** and **B**.

This sum will be another vector - let's call it **C**.

- Draw **A** with the correct size and angle
- Draw **B** with the correct size and angle, but such that **B**'s tail starts at the head of vector **A**
- The vector **C** is then represented by an arrow from the tail of **A** to the head of **B**.



Lecture 4: Vectors

3

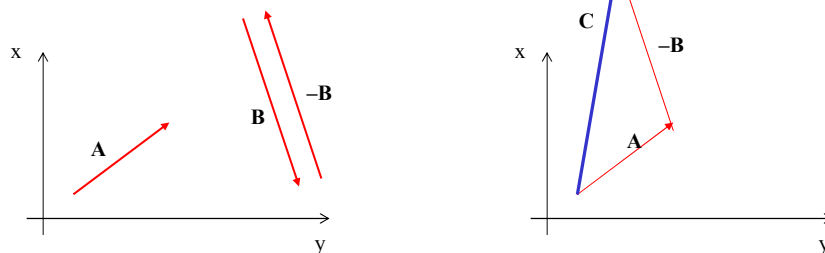
Subtracting Vectors Graphically

Say we wanted to find the difference of two vectors **A** and **B**.

This difference will be another vector - let's call it $C = A - B$.

This can be thought of as the sum $C = A + (-B)$.

- Draw **A** with the correct size and angle
- The vector **-B** is a vector with the same magnitude as **B**, but the opposite direction. Draw the vector **-B** with the correct size and angle, but such that **-B**'s tail starts at the head of vector **A**
- The vector **C** is then represented by an arrow from the tail of **A** to the head of **-B**.



Lecture 4: Vectors

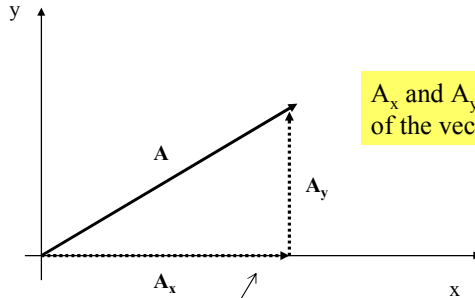
4

Resolving a vector into its components

Consider the vector A , which lies in the xy plane. It can be represented as the sum of two other vectors A_x and A_y :

- A_x is a vector which is parallel to the x -axis
- A_y is a vector which is parallel to the y -axis
- A_x and A_y form a right triangle

$$\vec{A} = \vec{A}_x + \vec{A}_y$$



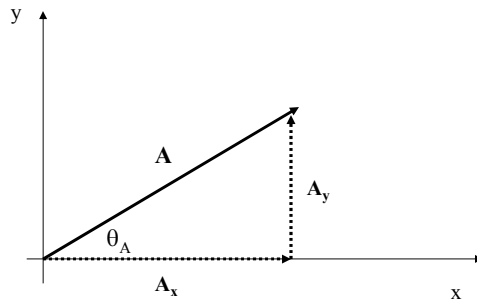
A_x and A_y are called the x and y components of the vector A .

This is a right angle

Lecture 4: Vectors

5

Components of Vectors



Note: A , A_x and A_y form a right triangle.

Given the components A_x and A_y , can find the magnitude and direction of the vector A :

$$|A| = \sqrt{A_x^2 + A_y^2}$$

$$\theta_A = \tan^{-1}\left(\frac{A_y}{A_x}\right)$$

Given the magnitude and direction of A , can find the components A_x and A_y :

$$A_x = A \cos(\theta)$$

$$A_y = A \sin(\theta)$$

Lecture 4: Vectors

6

Resolving a vector into its components

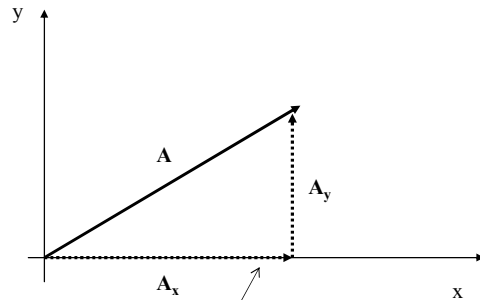
Another way of representing a vector using components uses *unit vectors*:

\hat{i} is parallel to the x - axis

\hat{j} is parallel to the y - axis

\hat{k} is parallel to the z - axis

Unit vectors don't have dimensions or units, they only point in a given direction.



This is a right angle

$$\vec{A}_x = A_x \hat{i}$$

$$\vec{A}_y = A_y \hat{j}$$

$$\vec{A} = A_x \hat{i} + A_y \hat{j}$$

Adding Vectors by Components

Say we want to find: $\vec{C} = \vec{A} + \vec{B}$

Let: $\vec{A} = A_x \hat{i} + A_y \hat{j}$ and $\vec{B} = B_x \hat{i} + B_y \hat{j}$

$$\vec{C} = \vec{A} + \vec{B}$$

Then

$$\vec{C} = (A_x \hat{i} + A_y \hat{j}) + (B_x \hat{i} + B_y \hat{j})$$

$$\vec{C} = (A_x + B_x) \hat{i} + (A_y + B_y) \hat{j}$$

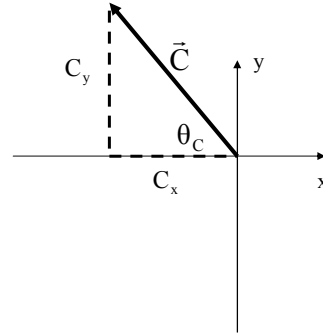
$$\vec{C} = C_x \hat{i} + C_y \hat{j}$$

where $C_x = A_x + B_x$
and $C_y = A_y + B_y$

Example: If $\vec{A} = (2\text{m})\hat{i} + (3\text{m})\hat{j}$ and $\vec{B} = (-8\text{m})\hat{i} + (7\text{m})\hat{j}$

What is: $\vec{C} = \vec{A} + \vec{B}$

Answer: $\vec{C} = (2\text{m} - 8\text{m})\hat{i} + (3\text{m} + 7\text{m})\hat{j}$
 $\vec{C} = -6\text{m}\hat{i} + 10\text{m}\hat{j}$



Magnitude of C:

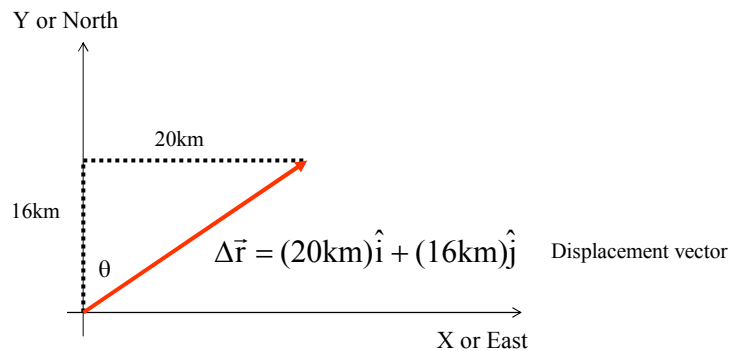
$$|C| = \sqrt{C_x^2 + C_y^2} = \sqrt{36 + 100} = \sqrt{136} = 12\text{m}$$

Direction of C:

$$\theta_C = \tan^{-1}\left(\frac{|C_y|}{|C_x|}\right) = \tan^{-1}\left(\frac{10}{6}\right) = 59^\circ \text{ Above the negative x-axis}$$

Example: A car travels 16km north, and then 20km east. The total travel time is 0.75h.

What is the average speed? $S_{\text{avg}} = \frac{\text{Total Distance}}{\text{Time Int}} = \frac{16 + 20}{0.75} = 48\text{km/h}$



Example: Continued

What is the average velocity? Remember that velocity is a vector!

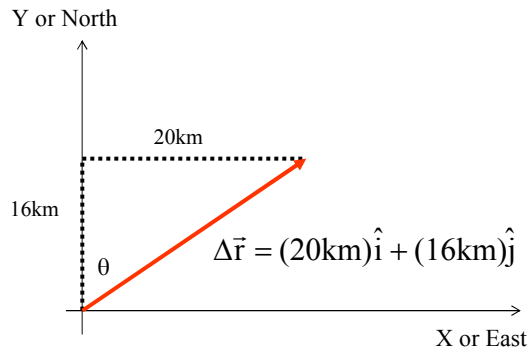
$$\vec{v}_{\text{avg}} = \frac{\Delta \vec{r}}{\Delta t}$$

$$v_{\text{avg},x} = \frac{\Delta x}{\Delta t} = \frac{20\text{km}}{0.75\text{h}} = 27 \text{ km/h}$$

$$v_{\text{avg},y} = \frac{\Delta y}{\Delta t} = \frac{16\text{km}}{0.75\text{h}} = 21 \text{ km/h}$$

$$|\vec{v}_{\text{avg}}| = \sqrt{v_{\text{avg},x}^2 + v_{\text{avg},y}^2} = 34 \text{ km/h}$$

$$\theta = \tan^{-1}\left(\frac{v_{\text{avg},y}}{v_{\text{avg},x}}\right) = 39^\circ$$



Is there another way to get the magnitude and direction of the average velocity?