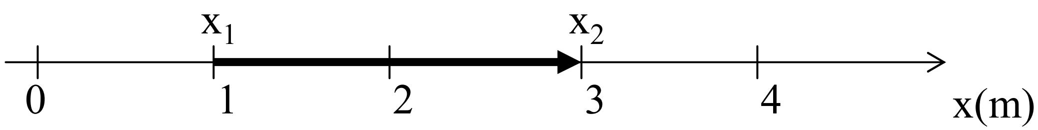
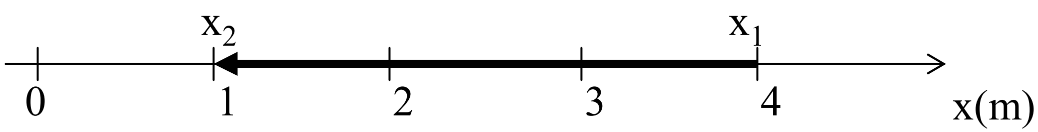
Vectors

Vector: A quantity that has both magnitude and direction.

- Displacement, velocity, and acceleration are examples of vectors
- In 1-dimension, the direction is specified by a +/- sign

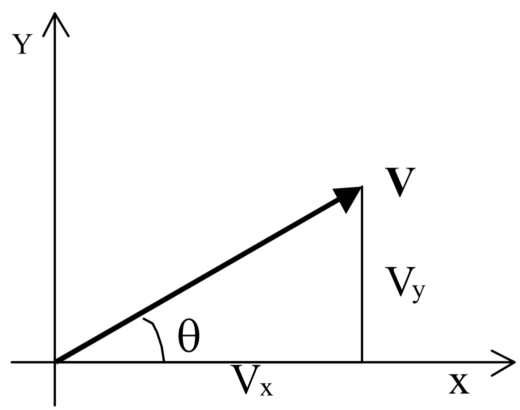


$$\Delta x = +2\text{m}$$



$$\Delta x = -3\text{m}$$

- In 2-dimensions, the direction is specified by an angle relative to one axis (usually the x-axis)

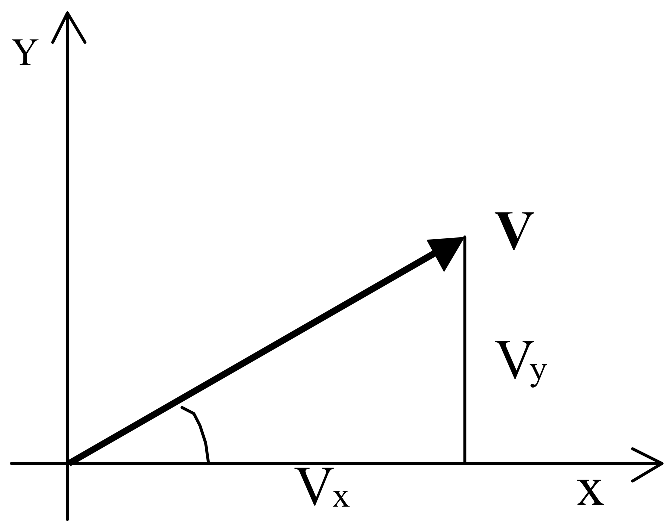


A vector is drawn using an Arrow.

- The angle relative to the x-axis gives the direction
- The length of the arrow gives the magnitude of the vector

Notation:

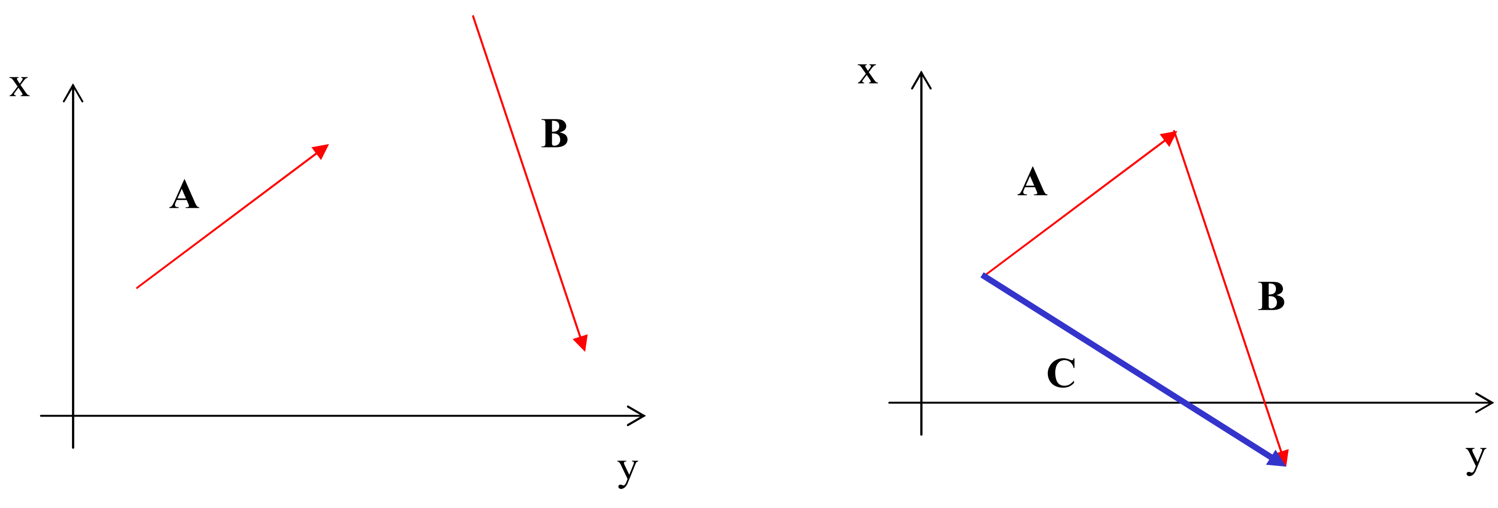
- In the book, the vector V is represented using boldface: **V**
- In class, and in notes, it is easier to represent a vector by writing a symbol with an arrow over it. So the vector V is represented as: $\vec{V}$



Adding Vectors Graphically

Say we wanted to find the sum of two vectors **A** and **B**.

This sum will be another vector - let's call it **C**.
- Draw **A** with the correct size and angle
- Draw **B** with the correct size and angle, but such that **B**'s tail starts at the head of vector **A**
- The vector **C** is then represented by an arrow from the tail of **A** to the head of **B**.



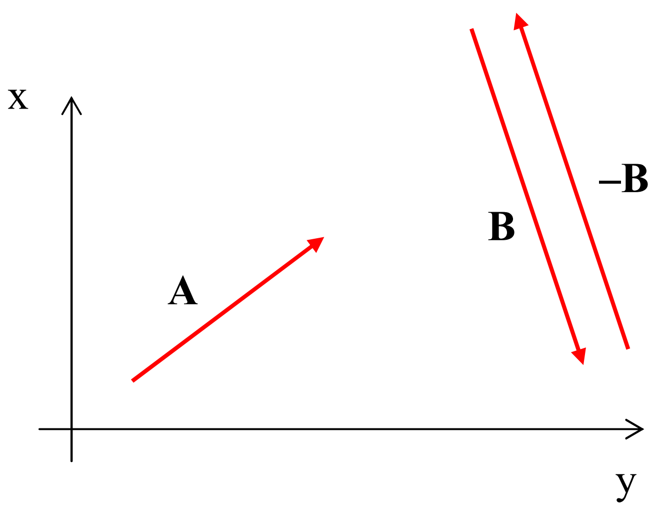
Subtracting Vectors Graphically

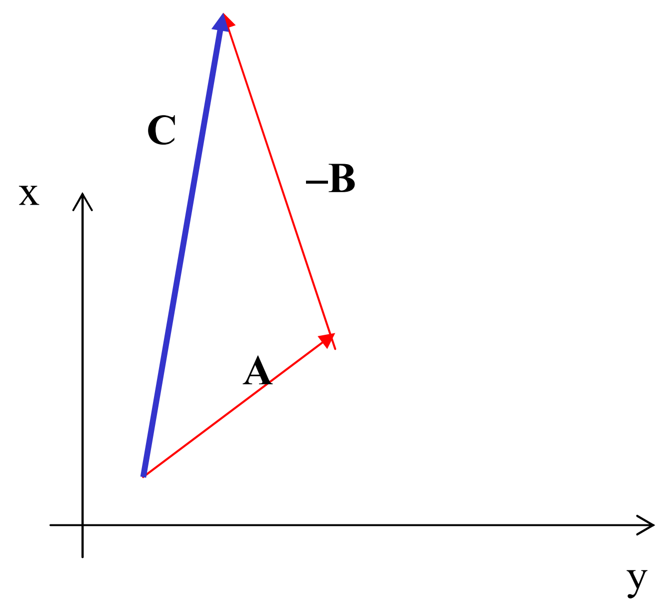
Say we wanted to find the difference of two vectors **A** and **B**.

This difference will be another vector - let's call it **C** = **A** - **B**.

This can be thought of as the sum **C** = **A** + **(-B).**

- Draw **A** with the correct size and angle
- The vector **-B** is a vector with the same magnitude as **B,** but the opposite direction. Draw the vector **-B** with the correct size and angle, but such that **-B**'s tail starts at the head of vector **A**
- The vector **C** is then represented by an arrow from the tail of **A** to the head of **-B**.



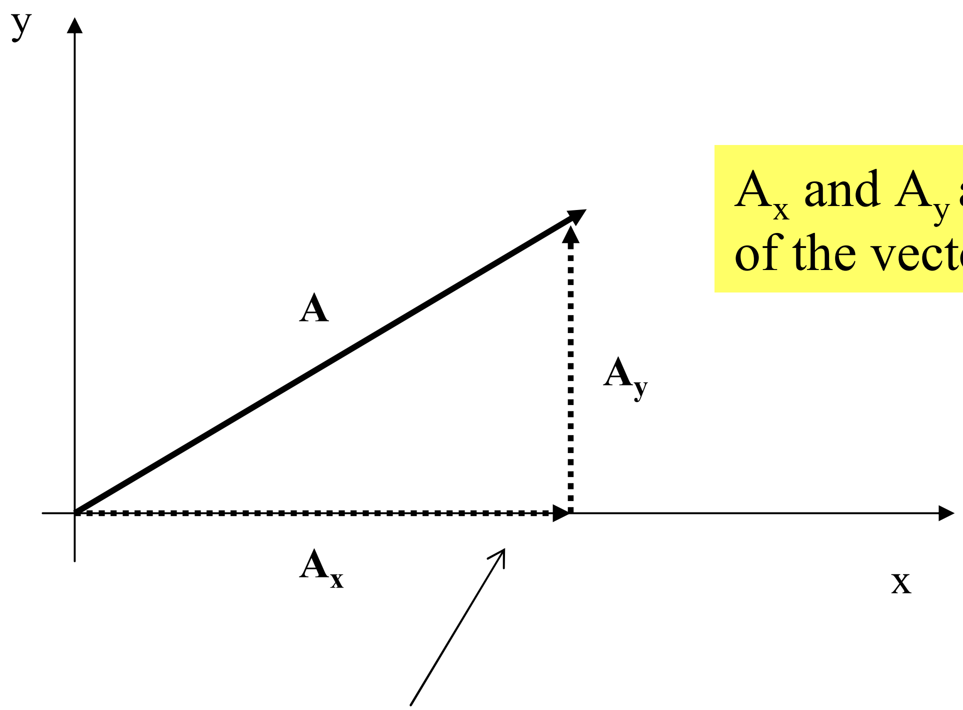


Resolving a vector into its components

Consider the vector A, which lies in the xy plane. It can be represented as the sum of two other vectors A_x and A_y :

- A_x is a vector which is parallel to the x-axis
- A_y is a vector which is parallel to the y-axis
- A_x and A_y form a right triangle

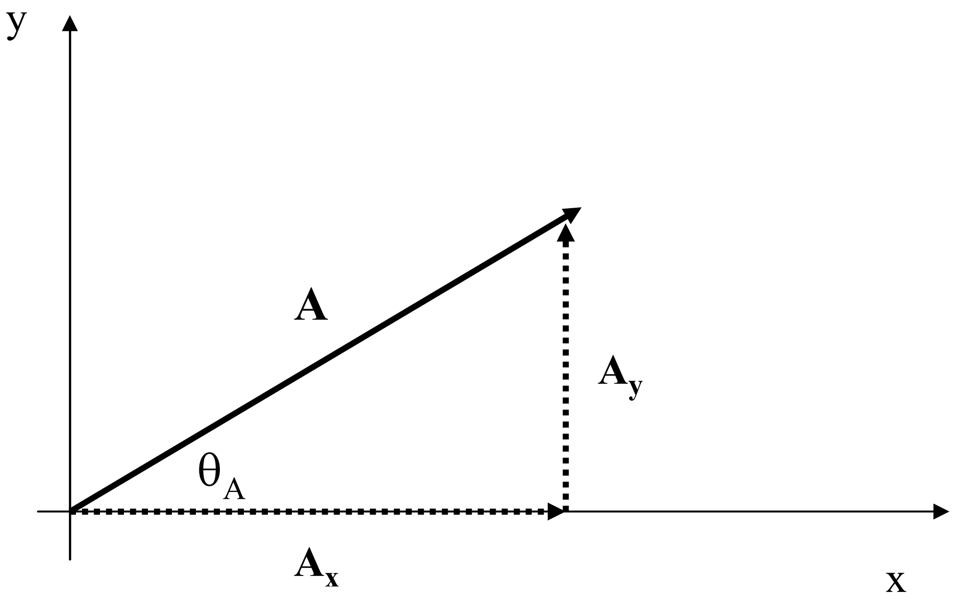
$$\vec{A} = \vec{A}_x + \vec{A}_y$$



This is a right angle

A_x and A_y are called the x and y components of the vector A.

Components of Vectors



Note: **A**, A_x and A_y form a right triangle.

Given the components A_x and A_y, can find the magnitude and direction of the vector A:

$$|A| = \sqrt{A_x^2 + A_y^2}$$

$$\theta_A = \tan^{-1}\left(\frac{A_y}{A_x}\right)$$

Given the magnitude and direction of **A**, can find the components A_x and A_y:

$$A_x = A\cos(\theta)$$

$$A_y = A\sin(\theta)$$

Resolving a vector into its components

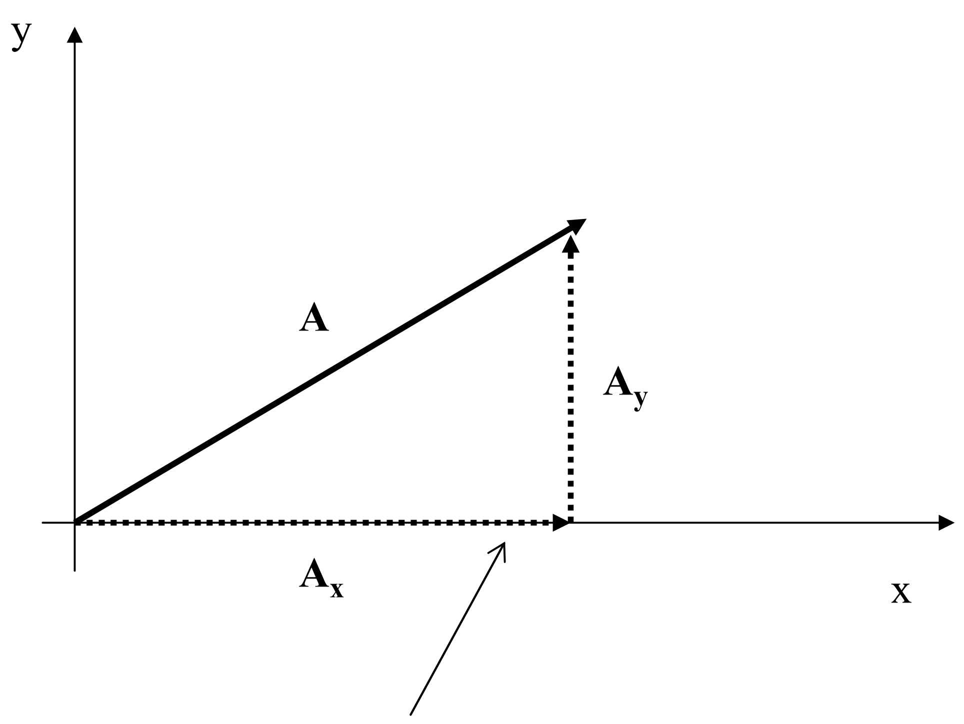
Another way of representing a vector using components uses *unit vectors*:

$\hat{i}$ is parallel to the x - axis

$\hat{j}$ is parallel to the y - axis

$\hat{k}$ is parallel to the z - axis

Unit vectors don't have dimensions or units, they only point in a given direction.



This is a right angle

$$\vec{A}_x = A_x\hat{i}$$

$$\vec{A}_y = A_y\hat{j}$$

$$\vec{A} = A_x\hat{i} + A_y\hat{j}$$

Adding Vectors by Components

Say we want to find: $\vec{C} = \vec{A} + \vec{B}$

Let: $\vec{A} = A_x\hat{i} + A_y\hat{j}$ and $\vec{B} = B_x\hat{i} + B_y\hat{j}$

Then

$$\vec{C} = \vec{A} + \vec{B}$$

$$\vec{C} = \left(A_x\hat{i} + A_y\hat{j}\right) + \left(B_x\hat{i} + B_y\hat{j}\right)$$

$$\vec{C} = \left(A_x + B_x\right)\hat{i} + \left(A_y + B_y\right)\hat{j}$$

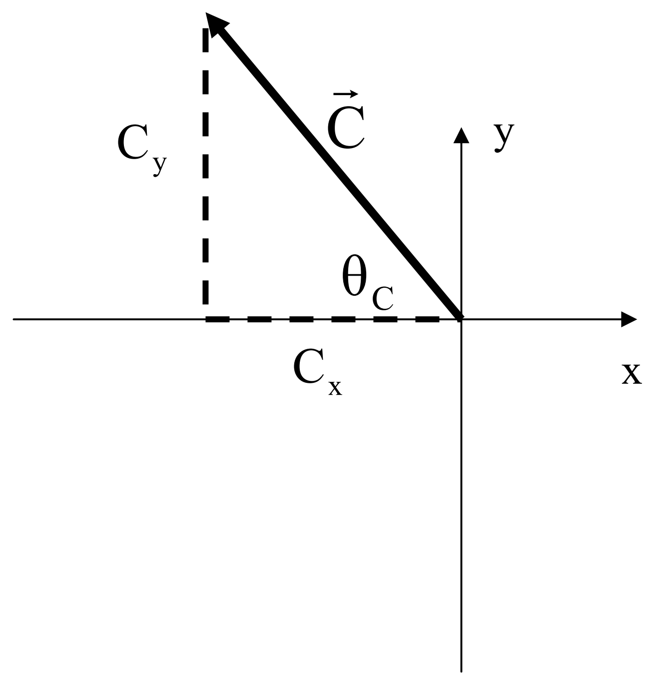
$$\vec{C} = C_x\hat{i} + C_y\hat{j}$$

where $C_x = A_x + B_x$

and $C_y = A_y + B_y$

Example: If $\vec{A} = (2m)\hat{i} + (3m)\hat{j}$ and $\vec{B} = (-8m)\hat{i} + (7m)\hat{j}$

What is: $\vec{C} = \vec{A} + \vec{B}$



Answer:

$$\vec{C} = (2m - 8m)\hat{i} + (3m + 7m)\hat{j}$$
$$\vec{C} = -6m\hat{i} + 10m\hat{j}$$

Magnitude of **C:**

$$|C| = \sqrt{C_x^2 + C_y^2} = \sqrt{36 + 100} = \sqrt{136} = 12m$$

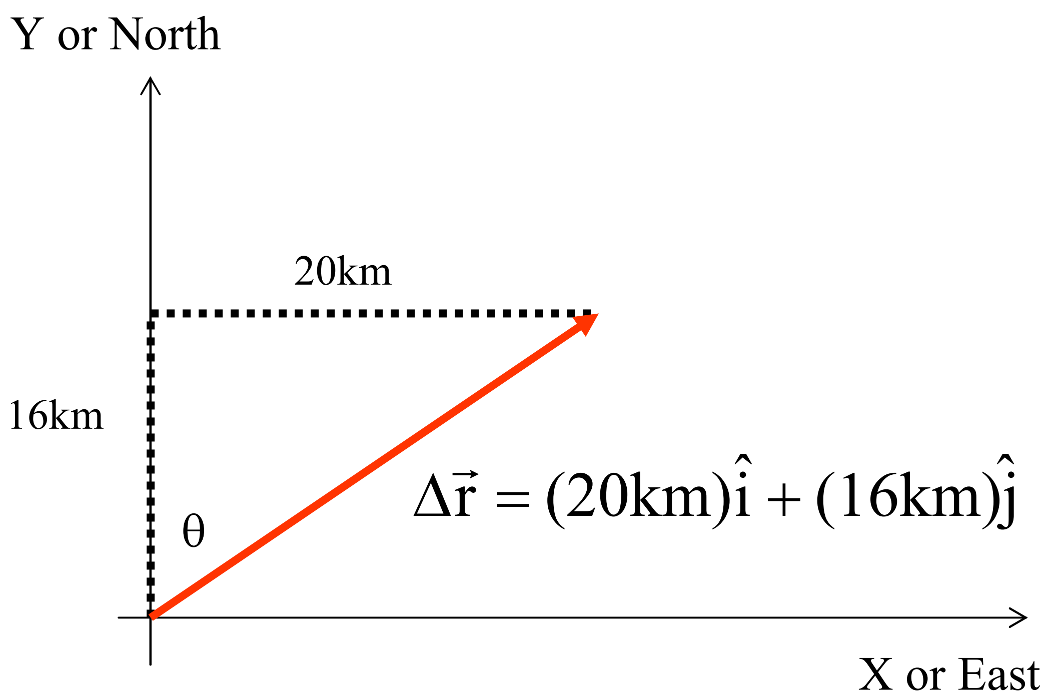
Direction of **C:**

$$\theta_C = \tan^{-1}\left(\frac{|C_y|}{|C_x|}\right) = \tan^{-1}\left(\frac{10}{6}\right) = 59^0$$ Above the negative x-axis

Example: A car travels 16km north, and then 20km east. The total travel time is 0.75h.

What is the average speed?

$$S_{avg} = \frac{\text{Total Distance}}{\text{Time Int}} = \frac{16+20}{0.75} = 48\text{km/h}$$



$$\Delta\vec{r} = (20\text{km})\hat{i} + (16\text{km})\hat{j}$$

Displacement vector

Example: Continued

What is the average velocity? Remember that velocity is a vector!

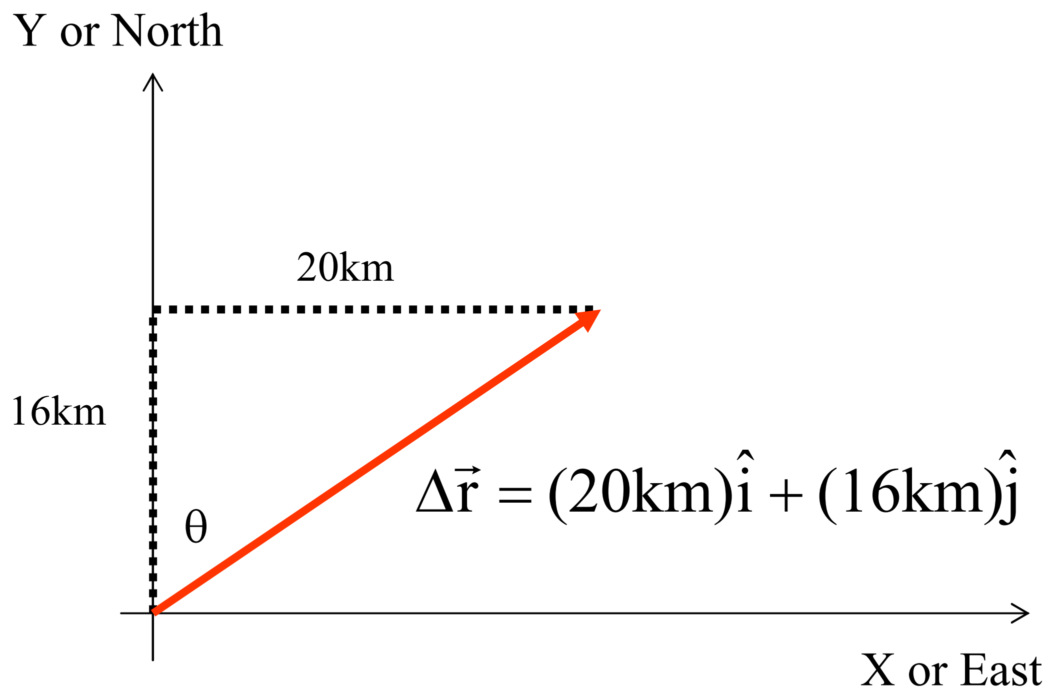
$$\vec{v}_{avg} = \frac{\Delta\vec{r}}{\Delta t}$$

$$v_{avg,x} = \frac{\Delta x}{\Delta t} = \frac{20km}{0.75h} = 27\ km/h$$

$$v_{avg,y} = \frac{\Delta y}{\Delta t} = \frac{16km}{0.75h} = 21\ km/h$$

$$\left|\vec{v}_{avg}\right| = \sqrt{v_{avg,x}^2 + v_{avg,y}^2} = 34\ km/h$$

$$\theta = \tan^{-1}\left(\frac{v_{avg,y}}{v_{avg,x}}\right) = 39^o$$



$$\Delta\vec{r} = (20km)\hat{i} + (16km)\hat{j}$$

Is there another way to get the magnitude and direction of the average velocity?