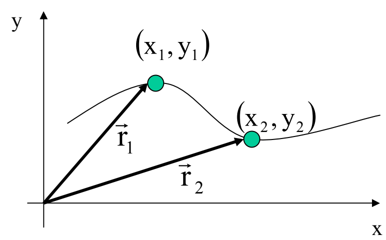
General Motion in 2 or 3 Dimensions

Consider 2-dimensions first (easy to generalize to 3-D)



Path of particle as a function of time = r(t)

The vector $\mathbf{r_1}$ tells you where the object is with respect to the origin (x=0,y=0) at time t_1.

As the object moves, the position changes from $\mathbf{r_1}$ to $\mathbf{r_2}$

Displacement:

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1 = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j}$$

$$\text{or} \quad \Delta\vec{r} = \Delta x\,\hat{i} + \Delta y\,\hat{j}$$

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Just as in 1-D, we can define an average velocity:

$$\vec{v}_{avg} = \frac{\Delta\vec{r}}{\Delta t} = \frac{\Delta x}{\Delta t}\hat{i} + \frac{\Delta y}{\Delta t}\hat{j} = v_{avg,x}\hat{i} + v_{avg,y}\hat{j}$$

an instantaneous velocity:

$$\vec{v}(t) = \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t} = v_x(t)\,\hat{i} + v_y(t)\,\hat{j}$$

And acceleration:

$$\vec{a}_{avg} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\Delta v_x}{\Delta t}\hat{i} + \frac{\Delta v_y}{\Delta t}\hat{j} = a_{avg,x}\hat{i} + a_{avg,y}\hat{j}$$

$$\vec{a}(t) = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t} = a_x(t)\,\hat{i} + a_y(t)\,\hat{j}$$

Example: The position of an object with time is given by:

$$\vec{r}(t) = (45t)\,\hat{i} + \left(1.5 - 4.9t^2\right)\hat{j}$$

Note: The units of both quantities in ( ) are in m. This means 45 = 45m/s and 4.9 = 4.9m/s².

a) What is v(t)?

$$\vec{v}(t) = \frac{dx(t)}{dt}\,\hat{i} + \frac{dx(t)}{dt}\,\hat{j}$$

$$\vec{v}(t) = 45\,\hat{i} - 9.8t\,\hat{j}$$

Example: Continued

b) What is a(t)?

$$\vec{a}(t) = \frac{dv_x(t)}{dt}\,\hat{i} + \frac{dv_y(t)}{dt}\,\hat{j}$$

$$\vec{a}(t) = (0)\,\hat{i} - 9.8\hat{j}$$

$$\vec{a}(t) = -9.8\hat{j}$$

c) Draw a picture of the y vs x motion from t = 0s to t = 0.5s.

t(s)	r_x(m)	r_y(m)
0.0	0.0	1.50
0.1	4.5	1.45
0.3	13.5	1.06
0.5	22.5	0.28