How to solve problems with constant acceleration in 1 dimension:

1) **Don't Panic!**

2) **Draw a simple picture of the problem.**

3) **Write what you know**:

x_0 = initial position

x = position at the time t

v_0 = initial velocity (remember the sign!)

v = velocity at the time t

a = the acceleration (constant!)

t = the time t

4) **Pick the correct equation:**

$$x = x_0 + v_0 t + \frac{1}{2} a t^2$$

$$v = v_0 + at$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

Example: (CH2, Prob 44) You are going 85mi/h, and you suddenly see a state trooper. How fast can you get to the legal speed limit of 65mi/h if your brakes can decelerate your car at a maximum of $17 ft/s^2$?

Solution:

1) Write what you know:

$$x_0 = 0$$
$$x = ?$$
$$v_{x0} = \left(85.0\frac{mi}{h}\right)\left(\frac{5280ft}{1mi}\right)\left(\frac{1h}{3600s}\right) = 125\ ft/s$$
$$v_x = 65.0\frac{mi}{h} = 95.0\ ft/s$$
$$a_x = -17.0\ ft/s^2$$
$$t = ?$$

2) Which equation to use?

$$v_x = v_{x0} + a_x t$$
$$95.0 = 125 + (-17.0)t$$
$$t = \frac{95.0 - 125}{-17.0} = 1.80s$$

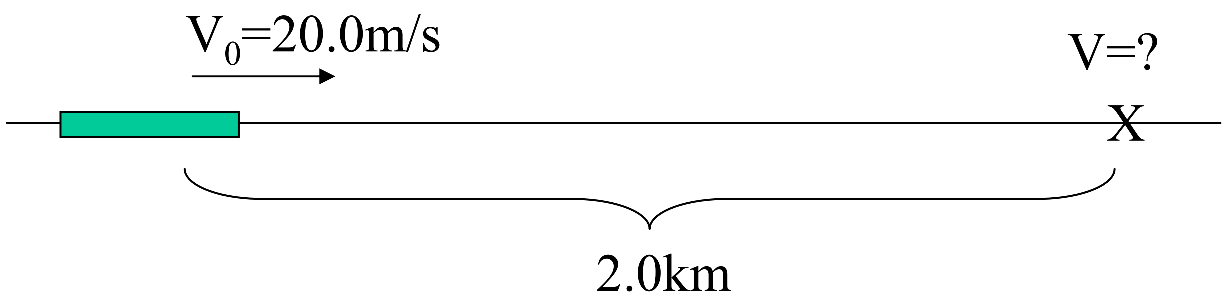
What does a deceleration of $17 ft/s^2$ mean?

Velocity is reduced by 17ft/s every second.

Example: An express train passes a station at 20.0m/s. The next station is 2.0km away and the train reaches it in 1.0min.

A) Did the train's velocity change?

B) If it did, assume the acceleration is constant, and determine the train's velocity at the second station.



1) If the train traveled at constant velocity (a=0), then

$$x = x_0 + v_{x0}t + \frac{1}{2}a_x t^2$$

$$x = v_{x0}t = (20.0\mathrm{m/s})(60\mathrm{s}) = 1200\mathrm{m} = 1.2\mathrm{km}$$

The train had to speed up to go 2.0km in 60s!

Continued===>

2) So now proceed as with a constant acceleration problem. Start by writing what you know:

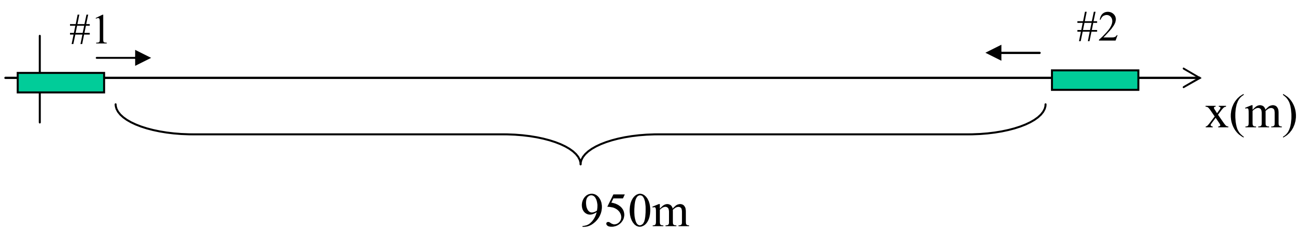
$$x_0 = 0$$
$$x = 2000\text{m}$$
$$v_{x0} = 20\text{m/s}$$
$$v_x = ?$$
$$a_x = ?$$
$$t = 60\text{s}$$

3) Next pick which equation(s) to use:

$$x = x_0 + v_{x0}t + \frac{1}{2}a_x t^2$$
$$\frac{1}{2}a_x t^2 = x - x_0 - v_{x0}t$$
$$a_x = \frac{2(x - x_0 - v_{x0}t)}{t^2} = \frac{2(2000 - 0 - 1200)}{3600} = 0.44\text{m/s}^2$$

$$v_x = v_{x0} + a_x t$$
$$v = 20\text{m/s} + 0.44(60)$$
$$v = 47\text{m/s}$$

Example: Two trains approach one another on the same track. The first is initially traveling at 72km/h, and the second is initially traveling at 144km/hr. When the trains are 950 m apart, they both hit their brakes. The brakes can decelerate each train at 1.0m/s². Will the two trains hit each other?



Train #1

$$x_{10} = 0$$
$$x_1 = ?$$
$$v_{10} = \left(72\frac{km}{h}\right)\left(\frac{1h}{3600s}\right)\left(\frac{1000m}{1km}\right) = 20m/s$$
$$v_1 = 0$$
$$a_1 = -1.0m/s^2$$
$$t = ?$$

Train #2

$$x_{20} = 950m$$
$$x_2 = ?$$
$$v_{20} = \left(-144\frac{km}{h}\right)\left(\frac{1h}{3600s}\right)\left(\frac{1000m}{1km}\right) = -40m/s$$
$$v_2 = 0$$
$$a_2 = +1.0m/s^2$$
$$t = ?$$

Example: Two trains continued...

Given the information for train 1, we can find its x position when it has finally stopped:

$$v_1^2 = v_{10}^2 + 2a_1(x_1 - x_{10})$$
$$0 = (20)^2 + 2(-1.0)(x_1)$$
$$x_1 = 200\text{m}$$

Given the information for train 2, we can also find its x position when it has finally stopped:

$$v_2^2 = v_{20}^2 + 2a_2(x_2 - x_{20})$$
$$0 = (40)^2 + 2(1.0)(x_2 - 950)$$
$$0 = 1600 + 2x_2 - 1900$$
$$x_2 = 150\text{m}$$

Since x_2 is less than x_1 the trains will hit!

Acceleration due to Gravity

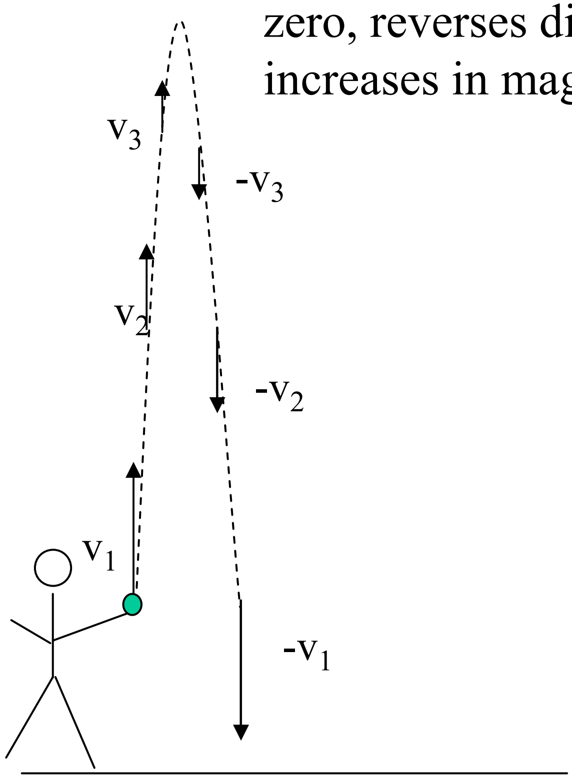
Throw an object straight up (or down), in the absence of air resistance.
Will always find that:

=> The object will accelerate ***downward*** at a constant rate

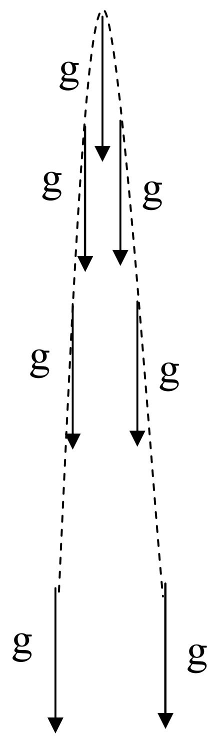
=> This rate is the free-fall acceleration, with a ***magnitude*** of $9.8 m/s^2$

=> We use the symbol **g** to represent this acceleration

Velocity first decreases, becomes zero, reverses direction, and then increases in magnitude



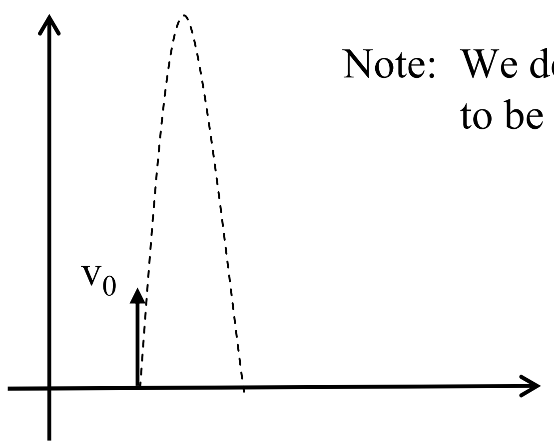
Acceleration remains constant during the whole motion, pointed downward



Example: A ball is thrown up in air, initial speed is 16m/s.
a) What will it's maximum height be?
b) How long will it take to get there?
c) How long will it take the ball to get to 10m?

Part a)

I) Draw a picture.



Note: We defined upward to be positive!

II) Write what you know

$$y_0 = 0$$
$$y = ?$$
$$v_{y0} = 16\text{m/s}$$
$$v_y = 0$$
$$a_y = -9.8\text{m/s}^2$$
$$t = ?$$

Example: Ball is thrown up in air continued

Part a) Continued

III) Which equation do we use? Since we don't know the time, we don't want to use either of the two equations which have time in them. We do know the initial and final velocity, and the initial position, so we can use the third equation.

$$v_y^2 = v_{y0}^2 + 2a_y(y - y_0)$$

$$0 = (16)^2 + 2(-9.8)y$$

$$y = \frac{256}{19.6} = 13\text{m}$$

Part b)

Pick the following eq:

$$v_y = v_{y0} + a_y t$$

$$0 = 16 - 9.8t$$

$$t = \frac{16}{9.8} = 1.6\text{s}$$

Aside to part b) How long did the ball take to fall?

I) Write what you know (ball is now in a different place)

$$y_0 = 13\text{m}$$
$$y = 0 \text{ (where it ends up)}$$
$$v_{y0} = 0 \text{ (at highest point)}$$
$$v_y = ?$$
$$a_y = -9.8\text{m/s}^2$$
$$t = ?$$

II) Which equation?

$$y = y_0 + v_{y0}t + \frac{1}{2}a_y t^2$$
$$0 = 13 + 0 + \frac{1}{2}(-9.8)t^2$$
$$t^2 = \frac{26}{9.8}$$
$$t = 1.6$$

Same time to fall as to rise!

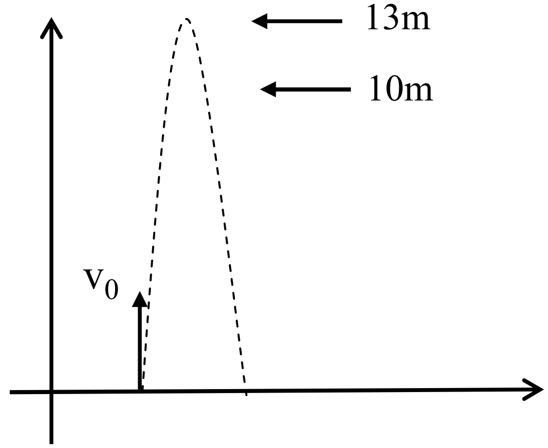
The velocity before hitting the ground:

$$v_y = v_{y0} + a_y t$$
$$v_y = 0 + (-9.8)(1.6)$$
$$v_y = -16\text{m/s}$$

Same magnitude as velocity when thrown up, but opposite in direction!

Part c) How long did the ball take to get to y=10m?

I) Draw a picture.



II) Write what you know

$y_0 = 0$

$y = 10m$

$v_{y0} = 16m/s$

$v_y = ?$

$a_y = -9.8m/s^2$

$t = ?$

II) Which equation?

$$y = y_0 + v_{y0}t + \frac{1}{2}a_y t^2$$

$$10 = 0 + 16t + \frac{1}{2}(-9.8)t^2$$

$$-4.9t^2 + 16t - 10 = 0$$

$$4.9t^2 - 16t + 10 = 0$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{16 \pm \sqrt{256 - 4(10)(4.9)}}{2(4.9)}$$

$t = 0.84$ or $2.4s$

(0.84s on the way up)

(2.4s on the way down)