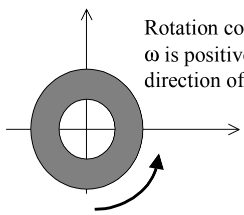
ω and α Are Vectors

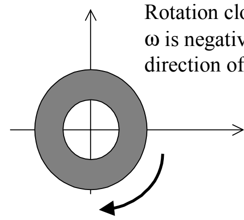
We have emphasized the fact that ω and α are the rotational equivalent of the translational velocity v and acceleration a. Another property they have in common with their translational counterparts is that **both ω and α are vectors**. We already know how to determine the magnitudes of ω and α, but how do we define their direction?

Direction of ω:

This will lie along the axis of rotation. For the figure below which is rotating in the plane of the paper, the axis of rotation comes straight out page. We define the positive direction of ω to be out of the page if the object is rotating counterclockwise. If the object is rotating clockwise, then ω is negative, and points into the page.



Rotation counterclockwise
ω is positive
direction of ω is out of page

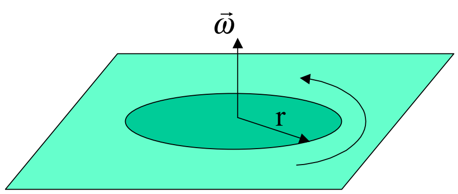


Rotation clockwise
ω is negative
direction of ω is into page

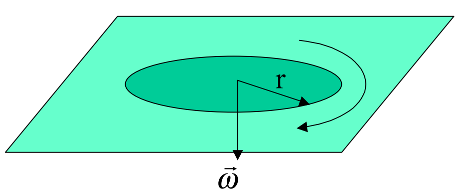
The Direction of ω From the Right Hand Rule

The direction of ω is can also be found by the **right hand rule**:

Let the fingers of your right hand curl around the object in the direction of rotation. Your thumb will then lie along the axis of rotation, pointing in the direction of ω.



Rotation counterclockwise
ω is positive
direction of ω is upwards

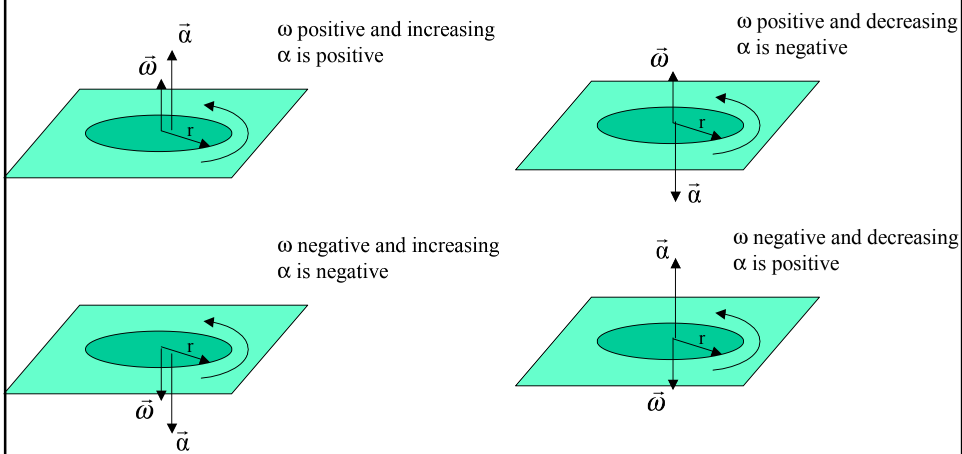


Rotation clockwise
ω is negative
direction of ω is downwards

The Direction of α

To get the direction of α, we need to remember: $\alpha = \frac{d\omega}{dt}$

If the axis of rotation is fixed, then ω lies along this axis, and therefore α will also lie along the axis.



The Direction of the Torque τ

Let's say that we have a rigid extend object attached to a fixed axis.

If a single force acts on that object in such a way as to cause it to rotate, then we say that the force imposes a torque on that object, and the magnitude of the torque is given by:

$$\tau = rF\sin\theta$$

Where r is the distance from the axis that the force is applied, and θ is the angle between the vectors r and F. However, we have also related torque to the moment of inertia of the object, and its angular acceleration:

$$\tau = I\alpha$$

Since the angular acceleration α is a vector, then **the torque τ is also a vector, and it points in the same direction as α**. But what if we did not know α, but we did know the direction of the vectors F and r. Could we determine the direction of the torque τ?

Torque and the Right Hand Rule

Torque is actually a vector - it is the result of the vector cross product of r and F:

$$\vec{\tau} = \vec{r} \times \vec{F}$$

The right hand of this equation is read: " r cross F "

The magnitude of the vector cross product of r and F is rFsinθ, which is exactly what we stated earlier for the magnitude of the torque:

$$\tau = rF\sin\theta$$

The direction of the cross product of r and F, and therefore of the torque τ, is also given by the right hand rule:

Let the fingers of your right hand point along the direction of r (from the axis to where the force is applied). Then curl your fingers in the direction of the force F (you may have to flip your hand upside down to do this). Your thumb will then point in the direction of the vector torque.