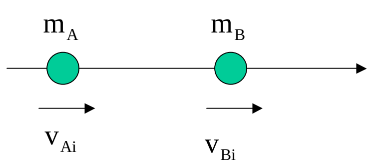
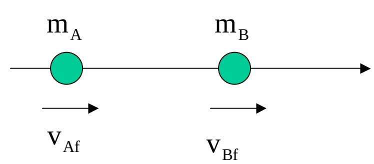
Elastic Collisions

Let's examine a particular kind of elastic collision: a head-on collision, in which all of the velocities lie along the same line. In this case we will choose the x-axis.

Before the collision:



After the collision:



Since no external forces act, momentum is conserved:

$$P_i = P_f$$
$$m_A v_{Ai} + m_B v_{Bi} = m_A v_{Af} + m_B v_{Bf}$$

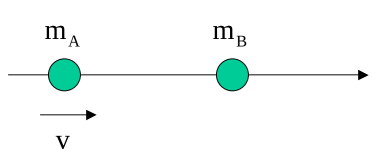
Since this is an elastic collision, kinetic energy is conserved:

$$K_i = K_f$$
$$\frac{1}{2} m_A v_{Ai}^2 + \frac{1}{2} m_B v_{Bi}^2 = \frac{1}{2} m_A v_{Af}^2 + \frac{1}{2} m_B v_{Bf}^2$$

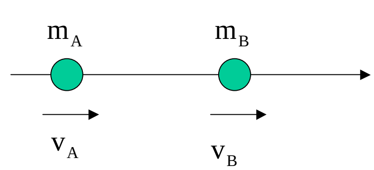
Elastic Collisions, where 1 object is initially at rest

The general solution to the above situation is pretty complicated. Let's look at a simpler situation, in which object B is initially at rest.

Before the collision:



After the collision:



In this case, the above two equations become:

$$m_A v = m_A v_A + m_B v_B \qquad \text{Equation 1}$$

$$\frac{1}{2} m_A v^2 = \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2 \qquad \text{Equation 2}$$

Elastic Collisions, Continued

Rewrite equation 1 as:

$$m_A v = m_A v_A + m_B v_B$$

$$\boxed{m_A\left(v - v_A\right) = m_B v_B}$$

Rewrite equation 2 as:

$$\frac{1}{2} m_A v^2 = \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2$$

$$m_A\left(v^2 - v_A^2\right) = m_B v_B^2$$

$$\boxed{m_A\left(v - v_A\right)\left(v + v_A\right) = m_B v_B^2}$$

Now take the two boxed equations, and divide the second by the first:

$$\frac{m_A\left(v - v_A\right)\left(v + v_A\right) = m_B v_B^2}{m_A\left(v - v_A\right) = m_B v_B} \longrightarrow v + v_A = v_B$$

Elastic Collisions, continued:

Don’t give up, we’re almost there!

By taking the first boxed equation: $m_A\left(v - v_A\right) = m_B v_B$

And the last equation: $v + v_A = v_B$

We can now solve for V_A and $V_{B:}$

$$v_A = \left(\frac{m_A - m_B}{m_A + m_B}\right) v \qquad v_B = \left(\frac{2m_A}{m_A + m_B}\right) v$$

Remember: These two equation are only valid for
• an elastic collision AND
• when particle B is at rest before the collision

Now that we have these two equations, how can we use them to make predictions?

Elastic Collisions, continued:

1) Particle B always moves in the positive direction.
2) Particle A will move in the positive direction if m_A is larger than m_B. Otherwise particle A will move in the negative direction after the collision (I.e. it will bounce backwards).
3) If $m_A >>> m_B$, then

$$v_A = \left(\frac{m_A - m_B}{m_A + m_B}\right) v \cong v \qquad v_B = \left(\frac{2m_A}{m_A + m_B}\right) v \cong 2v$$

Particle A moves on almost unaffected by the collision, while B moves off with almost twice the speed of A.

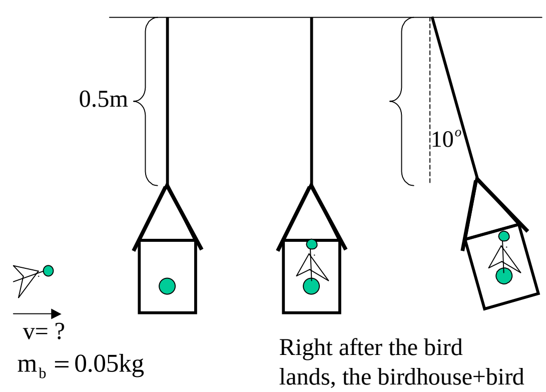
4) If $m_B >>> m_A$, then

$$v_A = \left(\frac{m_A - m_B}{m_A + m_B}\right) v \cong -v \qquad v_B = \left(\frac{2m_A}{m_A + m_B}\right) v \cong 0$$

Particle A bounces backwards after the collision with almost its original speed, while B is barely affected by the collision, moving slowly off in the original direction of particle A.

Inelastic Collision Example

A birdhouse of mass 0.2kg hangs from the branch of a tree by a 0.5m string. A 0.05kg bird lands at the birdhouse, and as a result, the birdhouse swings up to an angle of 10^o from the vertical. How fast was the bird flying just before landing at the birdhouse?



Right before the bird lands, the bird is flying with some speed v (which we want to know), and the birdhouse is at rest.

Right after the bird lands, the birdhouse+bird system has a final speed v_f

At this point, the birdhouse has swung up an angle of 10^o. The birdhouse+bird stop moving, and at this point the kinetic energy of the system is 0.

Inelastic Collision Example, Continued

There are really two problems here.
1) How fast is the “system” of birdhouse+bird moving right after the bird lands. This is a conservation of momentum problem.
2) Relating the kinetic energy of the system of birdhouse+bird to its potential energy.

Right before and after the collision (which is inelastic - do you know why?) momentum is conserved:

$$P_i = P_f \qquad P_i = m_b v + 0 \qquad P_f = (m_b + m_h)v_c$$

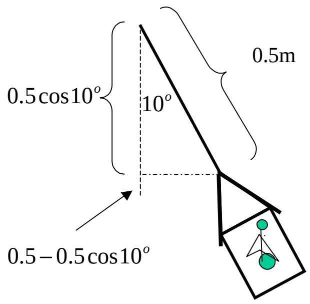
mass of bird $m_b = 0.05\text{kg}$
speed of bird before landing $v = ?$
mass of birdhouse $m_h = 0.20\text{kg}$
speed of birdhouse + bird, after landing $v_c = ?$

We know v, and the masses of the bird and birdhouse. How do we determine v_c, the speed of the bird+birdhouse right after the bird lands? We use **conservation of mechanical energy - only after the collision is over**.

After the bird lands on the birdhouse, the system of bird+birdhouse has a speed, and therefore an initial kinetic energy. The system then swings up like a pendulum, gaining potential energy and losing kinetic energy, until system has zero kinetic energy. During this time, **only conservative forces do work (gravity) and therefore, mechanical energy is conserved**.

Inelastic Collision Example, Continued

To determine the final potential energy, we need to know how high the birdhouse+bird system rose. We can get this from the information given, and we can then apply conservation of mechanical energy:



Initial

$$U_i = (m_b + m_h)gy_i = 0$$
$$K_i = \frac{1}{2}(m_b + m_h)v_c^2$$

Final

$$U_f = (m_b + m_h)gy_f$$
$$K_f = \frac{1}{2}mv_f^2 = 0$$

$$E_f = E_i$$
$$K_f + U_f = K_i + U_i$$
$$\frac{1}{2}(m_b + m_h)v_c^2 + 0 = 0 + (m_b + m_h)gy_f$$
$$v_c = \sqrt{gy_f}$$
$$v_c = \sqrt{(9.80)(0.0076)} = 0.27\text{m/s}$$

Inelastic Collision Example, Continued

Now that we have the speed of the bird+birdhouse right after the bird lands, we can now apply **conservation of momentum** (**<u>before and after the bird lands</u>**) to get the initial speed of the bird:

$$P_i = P_f$$

$$P_i = m_b v + 0 \qquad\qquad P_f = (m_b + m_h) v_c$$

$$P_i = P_f$$

$$m_b v = (m_b + m_h) v_c$$

$$v = \left(\frac{m_b + m_h}{m_b}\right) v_c$$

$$v = \left(\frac{0.25}{0.05}\right) 0.27 = 1.36 \text{m/s}$$