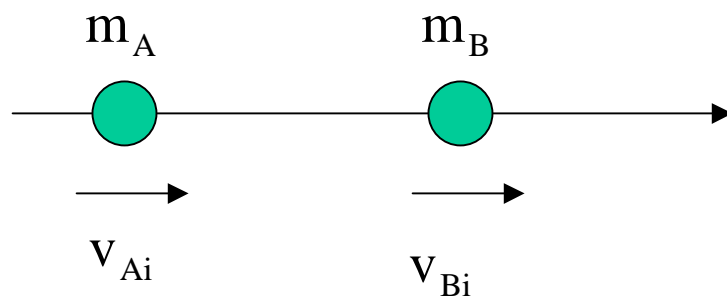


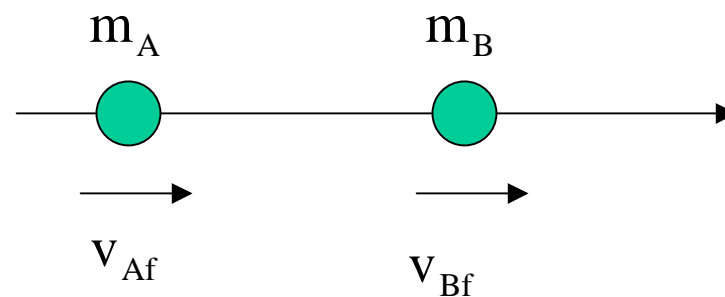
## Elastic Collisions

Let's examine a particular kind of elastic collision: a head-on collision, in which all of the velocities lie along the same line. In this case we will choose the x-axis.

Before the collision:



After the collision:



Since no external forces act, momentum is conserved:

$$P_i = P_f$$

$$m_A v_{Ai} + m_B v_{Bi} = m_A v_{Af} + m_B v_{Bf}$$

Since this is an elastic collision, kinetic energy is conserved:

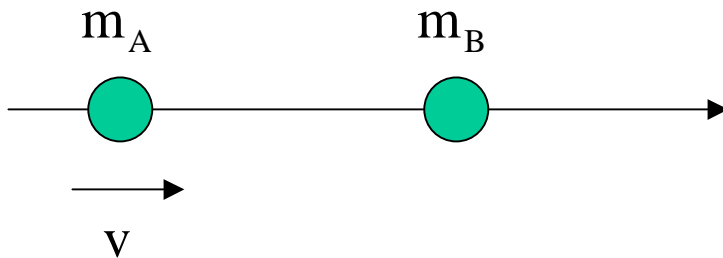
$$K_i = K_f$$

$$\frac{1}{2} m_A v_{Ai}^2 + \frac{1}{2} m_B v_{Bi}^2 = \frac{1}{2} m_A v_{Af}^2 + \frac{1}{2} m_B v_{Bf}^2$$

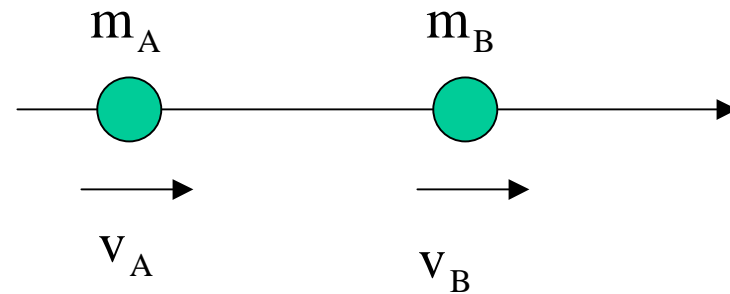
## Elastic Collisions, where 1 object is initially at rest

The general solution to the above situation is pretty complicated. Let's look at a simpler situation, in which object B is initially at rest.

Before the collision:



After the collision:



In this case, the above two equations become:

$$m_A v = m_A v_A + m_B v_B \quad \text{Equation 1}$$

$$\frac{1}{2} m_A v^2 = \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2 \quad \text{Equation 2}$$

## Elastic Collisions, Continued

Rewrite equation 1 as:

$$m_A v = m_A v_A + m_B v_B$$

$$m_A (v - v_A) = m_B v_B$$

Rewrite equation 2 as:

$$\frac{1}{2} m_A v^2 = \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2$$

$$m_A (v^2 - v_A^2) = m_B v_B^2$$

$$m_A (v - v_A)(v + v_A) = m_B v_B^2$$

Now take the two boxed equations, and divide the second by the first:

$$\frac{m_A (v - v_A)(v + v_A) = m_B v_B^2}{m_A (v - v_A) = m_B v_B} \quad \longrightarrow \quad v + v_A = v_B$$

## Elastic Collisions, continued:

Don't give up, we're almost there!

By taking the first boxed equation:

$$m_A (v - v_A) = m_B v_B$$

And the last equation:

$$v + v_A = v_B$$

We can now solve for  $v_A$  and  $v_B$ :

$$v_A = \left( \frac{m_A - m_B}{m_A + m_B} \right) v \quad v_B = \left( \frac{2m_A}{m_A + m_B} \right) v$$

Remember: These two equations are only valid for

- an elastic collision AND
- when particle B is at rest before the collision

Now that we have these two equations, how can we use them to make predictions?

## Elastic Collisions, continued:

- 1) Particle B always moves in the positive direction.
- 2) Particle A will move in the positive direction if  $m_A$  is larger than  $m_B$ . Otherwise particle A will move in the negative direction after the collision (I.e. it will bounce backwards).
- 3) If  $m_A \gg \gg m_B$ , then

$$v_A = \left( \frac{m_A - m_B}{m_A + m_B} \right) v \cong v \qquad v_B = \left( \frac{2m_A}{m_A + m_B} \right) v \cong 2v$$

Particle A moves on almost unaffected by the collision, while B moves off with almost twice the speed of A.

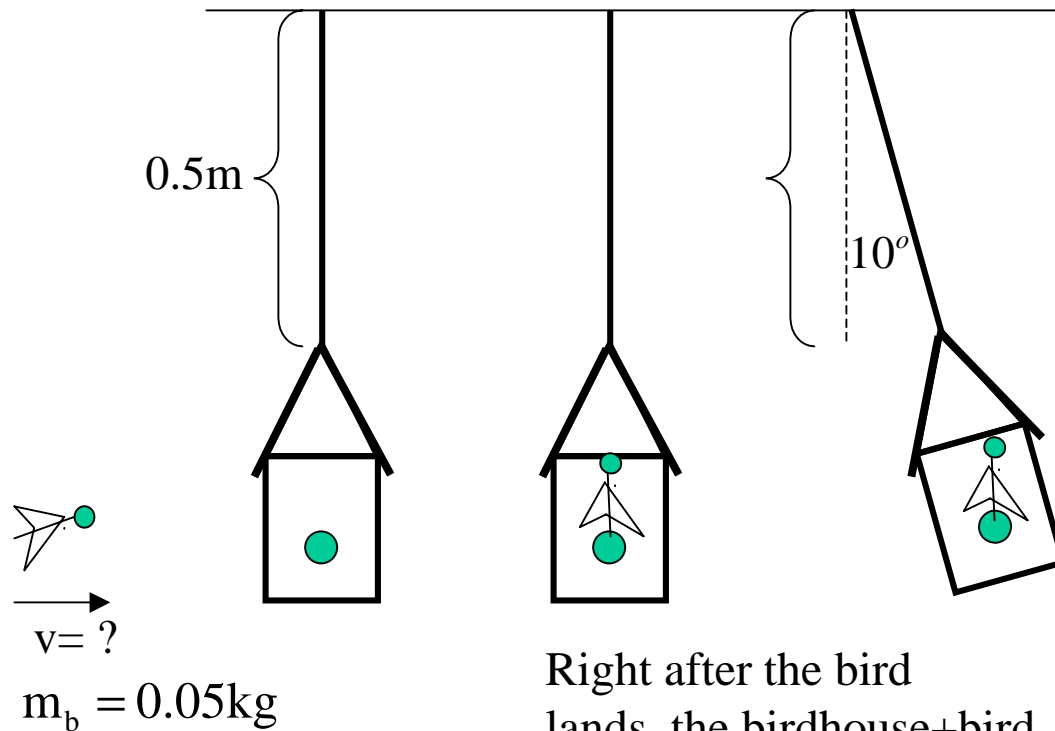
- 4) If  $m_B \gg \gg m_A$ , then

$$v_A = \left( \frac{m_A - m_B}{m_A + m_B} \right) v \cong -v \qquad v_B = \left( \frac{2m_A}{m_A + m_B} \right) v \cong 0$$

Particle A bounces backwards after the collision with almost its original speed, while B is barely affected by the collision, moving slowly off in the original direction of particle A.

## Inelastic Collision Example

A birdhouse of mass  $0.2\text{kg}$  hangs from the branch of a tree by a  $0.5\text{m}$  string. A  $0.05\text{kg}$  bird lands at the birdhouse, and as a result, the birdhouse swings up to an angle of  $10^\circ$  from the vertical. How fast was the bird flying just before landing at the birdhouse?



Right before the bird lands, the bird is flying with some speed  $v$  (which we want to know), and the birdhouse is at rest.

## Inelastic Collision Example, Continued

There are really two problems here.

- 1) How fast is the “system” of birdhouse+bird moving right after the bird lands. This is a conservation of momentum problem.
- 2) Relating the kinetic energy of the system of birdhouse+bird to its potential energy.

Right before and after the collision (which is inelastic - do you know why?) momentum is conserved:

$$P_i = P_f$$
$$P_i = m_b v + 0$$
$$P_f = (m_b + m_h) v_c$$

mass of bird  $m_b = 0.05\text{kg}$

speed of bird before landing  $v = ?$

mass of birdhouse  $m_h = 0.20\text{kg}$

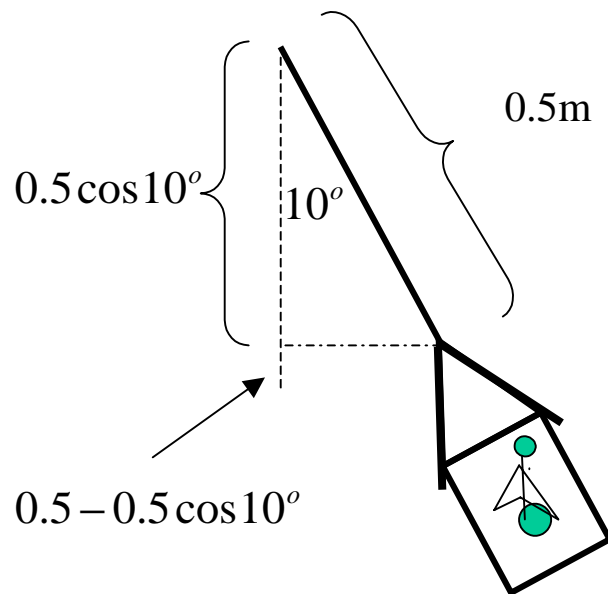
speed of birdhouse + bird, after landing  $v_c = ?$

We know  $v$ , and the masses of the bird and birdhouse. How do we determine  $v_c$ , the speed of the bird+birdhouse right after the bird lands? We use **conservation of mechanical energy - only after the collision is over.**

After the bird lands on the birdhouse, the system of bird+birdhouse has a speed, and therefore an initial kinetic energy. The system then swings up like a pendulum, gaining potential energy and losing kinetic energy, until system has zero kinetic energy. During this time, **only conservative forces do work (gravity) and therefore, mechanical energy is conserved.**

## Inelastic Collision Example, Continued

To determine the final potential energy, we need to know how high the birdhouse+bird system rose. We can get this from the information given, and we can then apply conservation of mechanical energy:



<u>Initial</u>	<u>Final</u>
$U_i = (m_b + m_h)gy_i = 0$	$U_f = (m_b + m_h)gy_f$
$K_i = \frac{1}{2}(m_b + m_h)v_c^2$	$K_f = \frac{1}{2}mv_f^2 = 0$

$$E_f = E_i$$

$$K_f + U_f = K_i + U_i$$

$$\frac{1}{2}(m_b + m_h)v_c^2 + 0 = 0 + (m_b + m_h)gy_f$$

$$v_c = \sqrt{gy_f}$$

$$v_c = \sqrt{(9.80)(0.0076)} = 0.27\text{m/s}$$

## Inelastic Collision Example, Continued

Now that we have the speed of the bird+birdhouse right after the bird lands, we can now apply **conservation of momentum** (before and after the bird lands) to get the initial speed of the bird:

$$P_i = P_f$$

$$P_i = m_b v + 0$$

$$P_f = (m_b + m_h)v_c$$

$$P_i = P_f$$

$$m_b v = (m_b + m_h)v_c$$

$$v = \left( \frac{m_b + m_h}{m_b} \right) v_c$$

$$v = \left( \frac{0.25}{0.05} \right) 0.27 = 1.36 \text{ m/s}$$