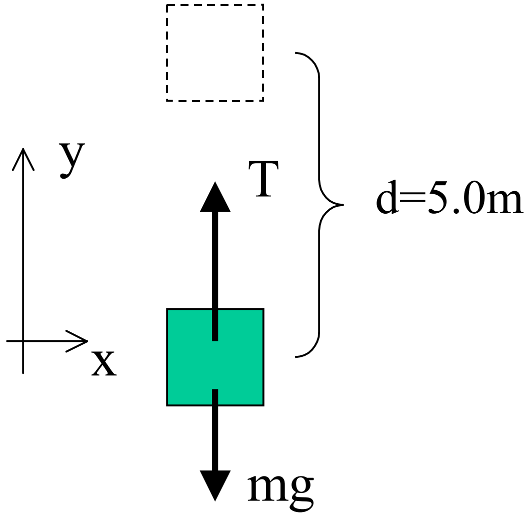
Conservative Forces

Let us lift an object of mass 10kg up a distance of 5.0m, and then lower back 5.0m, at constant speed. How much work is done by gravity?



In lifting the object, the work done is:

$$W_1 = (-mg)(d) = -(10.0\text{kg})(9.80\text{m/s}^2)(5.0\text{m}) = -490\text{J}$$

In lowering the object, the work done is:

$$W_2 = (-mg)(-d) = (10.0\text{kg})(9.80\text{m/s}^2)(5.0\text{m}) = 490\text{J}$$

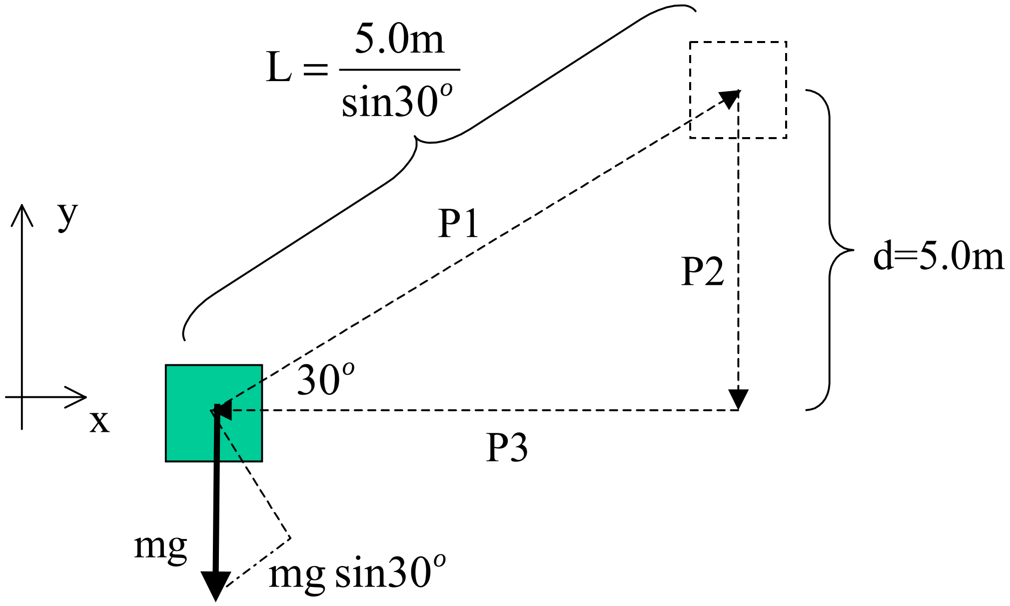
The total work done is then

$$W = W_1 + W_2 = -490\text{J} + 490\text{J} = 0$$

You should notice that no matter how high we lift the object, as long as we lower it back to where it started, the total work will equal zero.

Conservative Forces

What if instead the path of the object was as shown below.



A) The work done by gravity as the object is moved along path P1 is:

$$W_{P1} = (\vec{mg}) \cdot \vec{L} = (-\text{mg}\sin 30^o)\left(\frac{5.0}{\sin 30^o}\right) = -\text{mg}(5.0) = -490\text{J}$$

Conservative Forces

B) The work done by gravity as the object is moved along path P2 is:

$$W_{P2} = (-mg)(-d)$$
$$W_{P2} = 490\text{J}$$

C) The work done by gravity as the object is moved along path P3 is zero, since the force of gravity has no component along the displacement.

The total work done is then

$$W = W_{P1} + W_{P2} + W_{P3} = -490\text{J} + 490\text{J} + 0 = 0$$

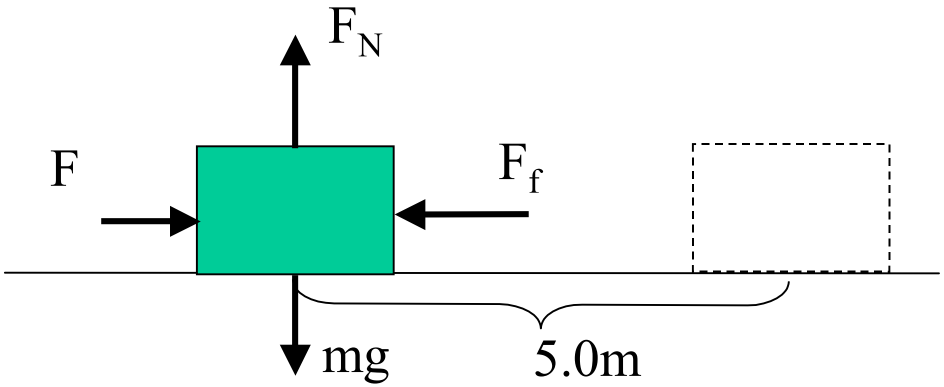
So we find that in the case of the two different paths taken, the total work is the same, as long as the paths have the same initial and final points. In addition, if the path is closed (meaning the initial and final points are the same), the total work done is zero.

Both of these statements are true for any conservative force.

Non-Conservative Forces

Push an object with mass 10.0kg a distance of 5.0m at constant speed along a rough surface, and then push it back. The coefficient of kinetic friction is 0.2.

How much work is done by friction for the entire path?



First we need to find the magnitude of the force of firction. We will use Newton's 2nd Law to determine this:

$$\sum F_y = ma_y = 0$$
$$F_N - mg = 0$$
$$F_N = mg$$

$$F_f = \mu_k F_N = \mu_k mg = 19.6\text{N}$$

Non-Conservative Forces

Work done by friction in moving the object forward 5.0m: (NOTE: This is not quite true-the work done by friction has some complicated features that we will come back to later.)

$$W = (-F_f)(5.0\text{m}) = -98\text{J}$$

Work done by friction in moving the object back 5.0m:

$$W = (F_f)(-5.0\text{m}) = -98\text{J}$$

The total work done is then

$$W = W_1 + W_2 = -98\text{J} - 98\text{J} = -196\text{J}$$

If we had pushed the object twice as far, and then back again, the work would double.

The work done by friction DOES depend on the path taken. This is true for any non-conservative force.

Conservative Forces and Potential Energy

Note the following things when we dealt with the force of gravity on the block:

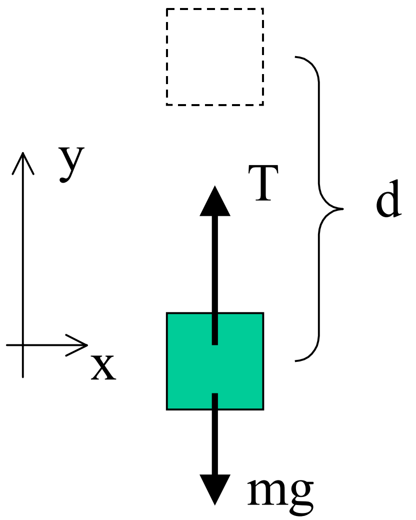
1) The block is actually **part of a system** - the block-earth system.
2) In raising the block, we are **changing the configuration** of the block-earth **system**.
3) In the case of gravity, the important change in the configuration was the change in the height of the block. No matter what path we choose, if the change in height is the same, then the amount of work that gravity does in moving the block to the new height is the same. When we move the block back to its original height, the work done by gravity is "returned".
4) This implies that the work done by the conservative force of gravity is somehow stored in the configuration of the block earth system. We say that the work is stored as the change in the potential energy in the system.

We will use the symbol U to refer to potential energy.

Work done by a conservative force is related to the **change** in potential energy of the system:

$$W = -\Delta U$$

Work and Gravitational Potential Energy



In raising the load a vertical height d, the work done by gravity is:

$$W = -mgd$$

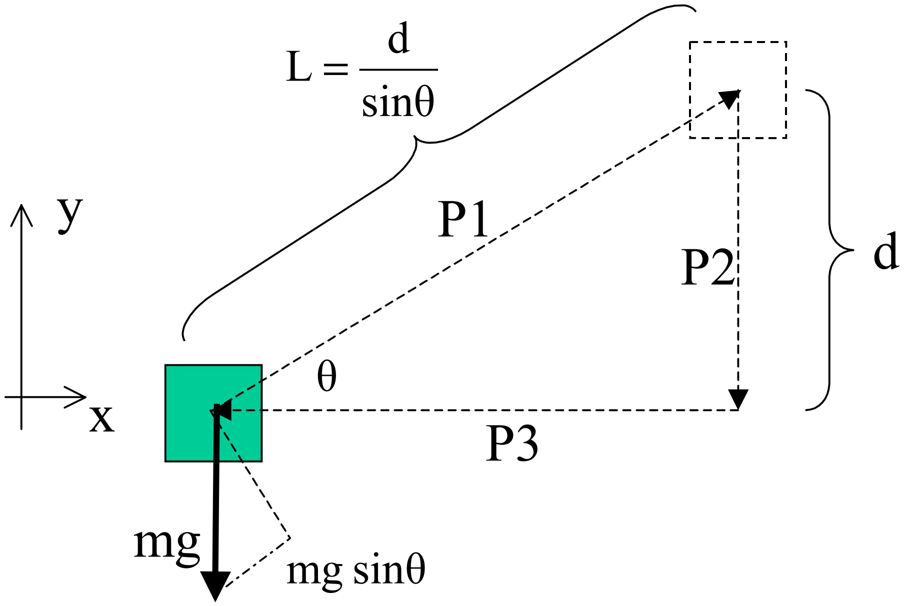
The change in potential energy is:

$$W = -\Delta U$$

$$\Delta U = -W = +mgd$$

The potential energy of the system has increased. The system now has potential to do work (for example, if we drop the load on something else).

Work and Gravitational Potential Energy



In raising the load in this manner, we find that the work done is:

$$W = -mg\sin\theta\,\frac{d}{\sin\theta} = -mgd$$

The change in potential energy is:

$$W = -\Delta U$$

$$\Delta U = -W = +mgd$$

The work and the potential energy only depend on the change in vertical height.

Gravitational Potential Energy

If we raise an object from a height y_i to a height y_f we find:

$$W = -mg(y_f - y_i) = -\Delta U$$
$$\Delta U = -W$$
$$U_f - U_i = mg(y_f - y_i)$$

We are free to define U_i as the **reference potential energy**, and usually we let this = 0. With this definition, we can define the gravitational potential energy of the system as:

$$U_{grav} = mgy$$

This is only valid if U=0 when y=0.

Elastic (or Spring) Potential Energy

Earlier, we found that the work done by a spring is given by:

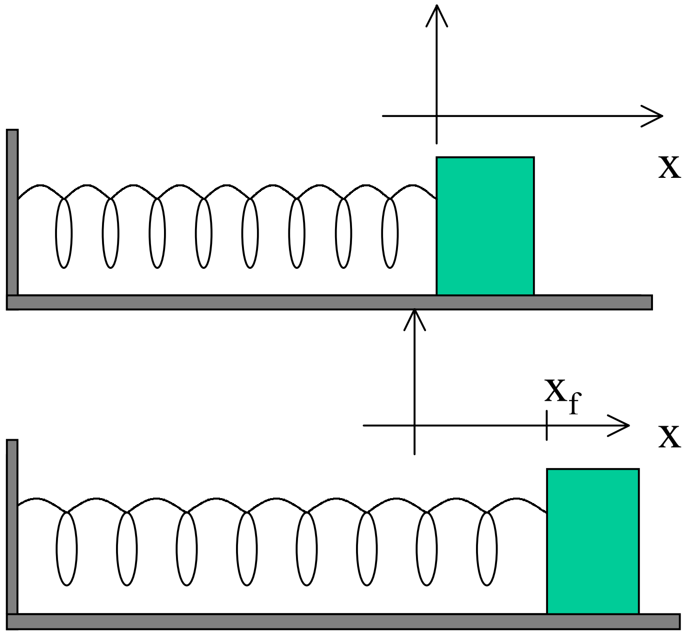
$$W = -\frac{1}{2}k\left(x_f^2 - x_i^2\right)$$

In a manner similar to that for the force of gravity, we can show that the **force of a spring is conservative**, and that we can define an elastic potential energy for springs:

$$W = -\frac{1}{2}k(x_f^2 - x_i^2) = -\Delta U$$

$$\Delta U = -W$$

$$U_f - U_i = \frac{1}{2}k(x_f^2 - x_i^2)$$



If we define the unstretched length of the spring as the configuration in which U=0, then we can define the elastic potential energy as:

$$U_{\text{spring}} = \frac{1}{2}kx^2$$

Conservation of Mechanical Energy

Let's consider a special case of a system in which only conservative forces act. When a conservative force does work on an object in this system:

1) Work and Potential Energy: $W = -\Delta U = -\left(U_f - U_i\right)$

2) The Work-Kinetic Energy Theorem: $W = \Delta K = K_f - K_i$

We can combine these two relationships:

$$\Delta K = -\Delta U$$

$$K_f - K_i = -\left(U_f - U_i\right)$$

$$K_f + U_f = K_i + U_i$$

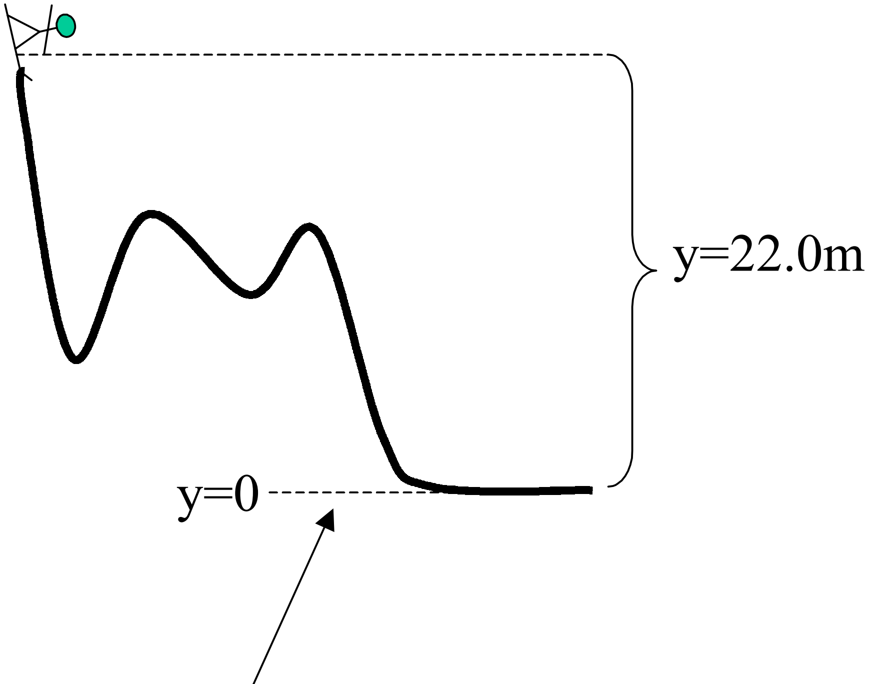
This says that when only conservative forces act, the initial kinetic energy plus the initial potential energy equals the final kinetic energy plus the final potential energy. We can define the sum of the kinetic and potential energies as the mechanical energy $E = K + U$

And we can then say that when only conservative forces act, the mechanical energy of the system does not change. This is the **Principle of Conservation of Mechanical Energy.**

$$\Delta E = \Delta K + \Delta U = 0$$

Example: Conservation of Mechanical Energy

An skier is at rest at the top of a slope which has a vertical drop of 22.0m. The slope has the complicated shape as show below. If the mass of the skier is 25.0kg, what is her speed when she reaches the flat portion of the slope at the bottom? Assume the slope is frictionless.



We will define this point as y=0, and therefore this is the point at which we also define U=0.

Since only conservative forces act, we know that the total mechanical energy of the system is conserved.

$$\Delta E = \Delta K + \Delta U = 0$$

or

$$K_f + U_f = K_i + U_i$$

Example(cont): Conservation of Mechanical Energy

Initial

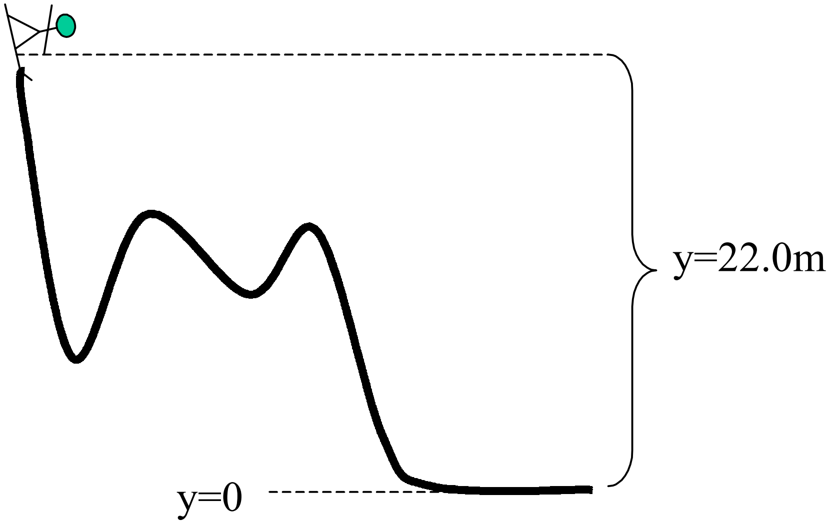
$$U_i = mgy_i = (25)(9.80)(22) = 5.39\times10^3$$

$$K_i = \frac{1}{2}mv_i^2 = 0$$

Final

$$U_f = mgy_f = 0$$

$$K_f = \frac{1}{2}mv_f^2$$



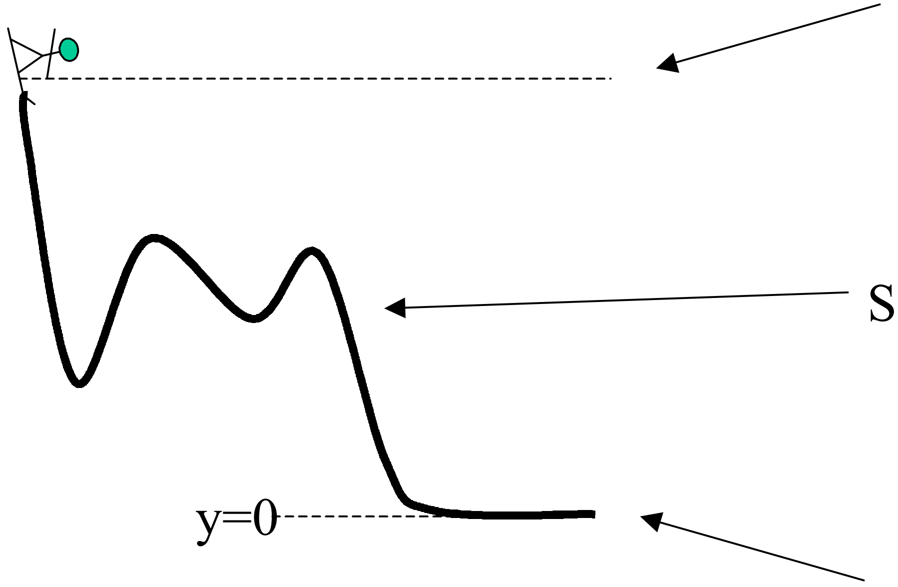
$$K_f + U_f = K_i + U_i$$

$$\frac{1}{2}mv_f^2 + 0 = 0 + mgy$$

$$v_f = \sqrt{2gy}$$

$$v_f = \sqrt{2(9.80)(22.0)} = 20.8\text{m/s}$$

More on Conservation of Mechanical Energy



All potential energy

$$E_i = K_i + U_i = mgy_i = 5.39\times10^3\,\mathrm{J}$$

Some potential and some kinetic energy:

$$E = K + U = 5.39\times10^3\,\mathrm{J}$$

All kinetic energy

$$E_f = K_f + U_f = \frac{1}{2}mv_f^2 = 5.39\times10^3\,\mathrm{J}$$

At various times in the motion of the skier, the energy may be all potential, all kinetic, or some combination of the two. However, as long as only conservative forces act, **the sum of kinetic and potential energy - the total mechanical energy - remains constant.**

More on Conservation of Mechanical Energy

Some important points about the last example:

1) Note that we could NOT use our equations of motion under constant acceleration to solve the last problem. Why? Because the acceleration is most definitely not constant!

2) Also note that the velocity at the beginning of the motion has both x and y components, while at the end of the motion it has on an x-component.

3) As in many problems we have done before, the mass drops out of the problem. This is one important reason why you should only substitute numbers in at the end of a calculation. If we had not done this, it would have been difficult to tell that the mass could have been doubled (or tripled, etc) and the result would be the same (for the final velocity).

Work Done By Non-Conservative Forces

So far we have dealt with problems in which the forces are all conservative. This is not always the case. If friction is present, or if an external force is applied to a system, both of which are non-conservative forces, then mechanical energy is NOT conserved. What do we do then?

1) When work is done by an applied force, acting on a system:

$$\Delta E = \Delta K + \Delta U = W_{app}$$

This says that the work done by an applied force can change the mechanical energy (either KE or PE or both) of a system.

2) When work is done by a kinetic frictional force $\mathbf{F_{fk}}$, acting on a system. An example would be a frictional force acting on a block sliding over a floor a distance d.:

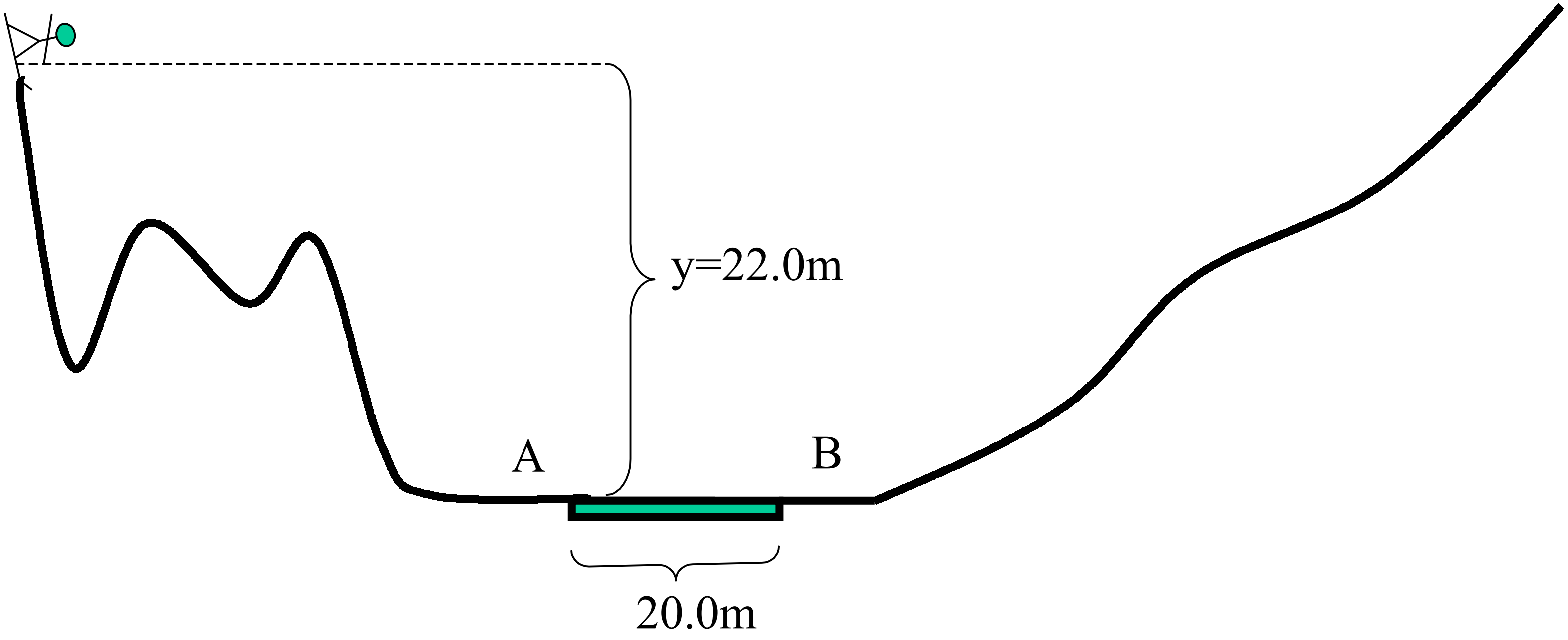
$$\Delta E = \Delta K + \Delta U = -F_{fk}d$$

This is the dissipated mechanical energy.

This term is NOT equal to the work done by the frictional force on the block. The work done by the frictional force is only that part of the energy which is transferred from the block to the floor. Since some of the dissipated energy remains with the block - as when it heats up, for example.

Work Done By Non-Conservative Forces

In the previous example of the skier, we assumed that the entire slope was frictionless. Imagine that at the bottom, the skier hits a flat stretch where the snow is melted. The coefficient of friction of this stretch is 0.2, and the patch is 20.0 m long. On the other side of the patch there is another slope. What vertical height does the skier reach on this second slope?



Previously, we determined that at point A, the skier's energy was all kinetic, and had the value:

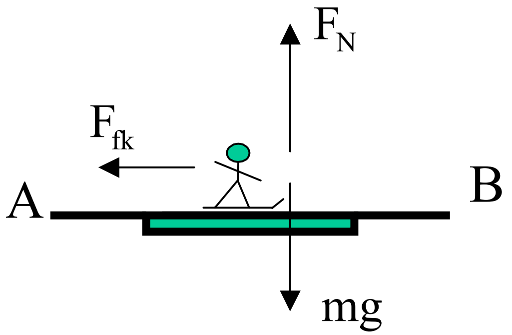
$$E_A = K_A + U_A = \frac{1}{2}mv_A^2 = 5.39\times10^3\,\mathrm{J}$$

Work Done By Non-Conservative Forces

As the skier goes from point A to point B, the mechanical energy is changed by the amount: $\Delta E = \Delta K + \Delta U = -F_{fk} d$

Since A and B are at the same height, $\Delta U=0$, $\Delta E = \Delta K = -F_{fk} d$

What is the force of friction? $F_{fk} = \mu_k F_N$



We need to get the normal force:

$$\sum F_y = ma_y = 0$$
$$F_N - mg = 0$$
$$F_N = mg$$

$$F_{fk} = \mu_k mg$$
$$F_{fk} = (0.2)(25.0)(9.80) = 49.0\text{N}$$

Work Done By Non-Conservative Forces

Now we can get the change in the mechanical energy of the system:

$$\Delta E = \Delta K = -F_{fk}d = -(49.0)(20.0)$$
$$\Delta E = \Delta K = -980J$$

And we can also get the kinetic energy of the skier at point B:

$$\Delta K = K_B - K_A = -980J$$
$$K_B = K_A - 980J = 5390 - 980$$
$$K_B = 4.41\times10^3 J$$

How high does the skier go? After traversing the bare patch, the skier is on frictionless snow again, and only conservative forces act.
Now we have conservation of mechanical energy again.

Initial

$$U_i = 0$$
$$K_i = K_B = 4.41\times10^3 J$$

Final

$$U_f = mgy_f$$
$$K_f = 0$$

$$K_f + U_f = K_i + U_i$$
$$0 + mgy_f = 4.41\times10^3 + 0$$
$$y_f = 18.0m$$