Work and Energy

We are now ready to develop some new ideas which describe the motion of objects. These ideas include:

- Kinetic energy
- Work
- Power
- Potential Energy
- Total Energy

Probably the most important idea is that of **Conservation of Energy**. This idea is important for a number of reasons:

1) **It gives us a powerful tool for analyzing the motion of particles or systems of particles.** Problems are often easier to solve using an "Energy" approach rather than a "Forces" approach.

2) **It is a fundamental statement about the universe, even more fundamental than Newton's Laws.** At high speeds, Newton's Laws need to be replaced by relativity. At small dimensions, Newton's Laws need to be replaced by quantum mechanics. But in both regions, the concept of Conservation of Energy is still valid.

Kinetic Energy

The first type of energy we will consider is called Kinetic energy, and it is associated with the motion of an object. The symbol for kinetic energy is K, and it is defined by the following relation:

$$K = \frac{1}{2} m v^2$$

The units of energy are Joules, written as J:

$$1\,\text{J} = 1\,\text{kg} \cdot \text{m}^2/\text{s}^2$$

Note:

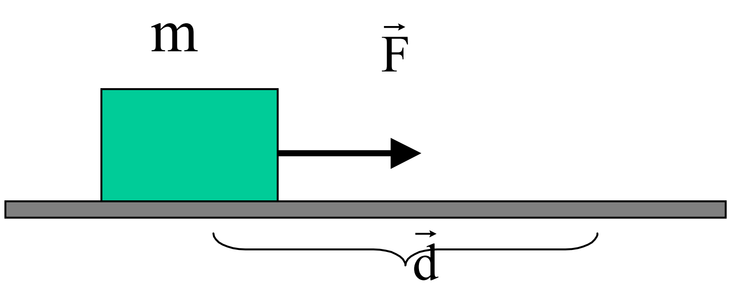
1) **The kinetic energy is directly proportional to the mass.** An object which is twice the mass of another, but moving at the same speed will have twice the kinetic energy of the other.
2) **The kinetic energy is directly proportional to the square of the velocity.** An object which has twice the speed of another object of the same mass, will have 4 times the kinetic energy of the other.
3) **The kinetic energy can never be negative**, since mass is always positive, and the square of the velocity is always positive.

Work

Why is work important? Because in any motion, the change in kinetic energy is equal to the total work done on it.

WORK: Work is energy transferred to or from an object by means of a force acting on that object.
Positive work occurs when energy is transferred to the object.
Negative work occurs when energy is transferred from the object.

Simple case: Single constant force acting on an object, causing it to undergo a displacement d. Define **work** done **by the force on the object** to be:

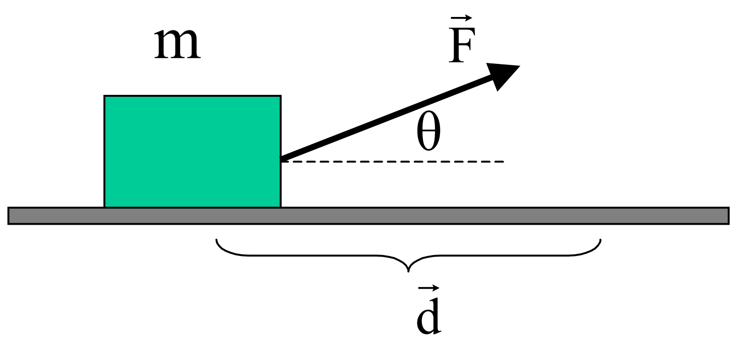


$$W = |\vec{F}||\vec{d}| = Fd$$

The units of work are the same as that of energy: Joules, written as J.

Work

What if there had been an angle between the force and the displacement?



We might expect that only the component of the force F which is along the displacement d does any work (and this is correct):

$$W = (F\cos\theta)d$$

NOTE: The only reason cos θ is in the above expression is because it gives us the component of the force along the displacement.

Important points about work:

- Work is a scalar quantity.
- Work is done by a force only if the body moves in such a way that the force has a component along the displacement.
- If the force has a component in the same direction as the displacement, the work done by the force, on the object, is positive
- If the force has a component in the opposite direction of the displacement, the work done by the force, on the object, is negative.

Work and Kinetic Energy

Imagine the above block initially had position x_i and velocity v_{xi}. The above force is applied and the object undergoes displacement d, such that its new position is x_f. If the force was constant, then the object had a constant acceleration a=(Fcosθ)/m, and therefore its velocity increased, to say, v_{xf}. We know from our study of motion under constant acceleration that:

$$x_0 = x_i$$
$$x = x_0$$
$$v_{x0} = v_i$$
$$v_x = v_f$$
$$a_x = \frac{F\cos\theta}{m}$$
$$t = ?$$

$$v_x^2 = v_{x0}^2 + 2a_x(x - x_0)$$

$$v_f^2 = v_i^2 + 2\left(\frac{F\cos\theta}{m}\right)d$$

Multiplying both sides of the last equation by m/2, and using the fact that **W=F cosθ d,** we find:

$$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = W$$

$$K_f - K_i = \Delta K = W$$

Work-Kinetic Energy Theorem:

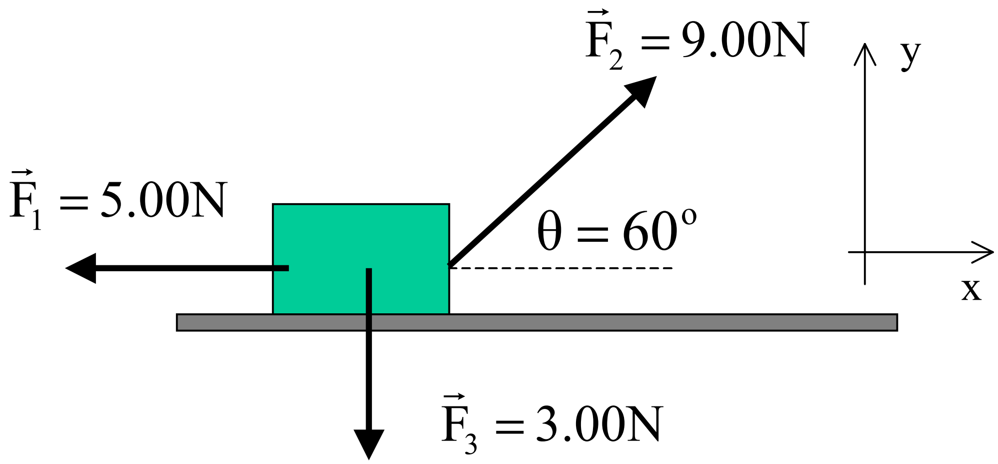
The work on an object done by a net External force is equal to the change in the kinetic energy of the object.

Example: Work and Kinetic Energy

Chapter 7, prob 16: 3 forces are applied to a trunk that moves to the left a distance of 3.00m over a frictionless floor, as shown below.

A) What is the net work done on the trunk by the 3 forces?

B) Does the kinetic energy of the trunk increase or decrease?



Example Continued: Work and Kinetic Energy

$$W = (F_1\cos\theta_1)d + (F_2\cos\theta_2)d + (F_3\cos\theta_3)d$$
$$W = (-5.0N)(1.0)(-3.00m) + (9.0N)(0.5)(-3.00m) + (3.0N)(0)(-3.00)$$
$$W = +15J - 13.5J = 1.5J$$

Since the displacement is only along the x-direction, we could also just calculate the net force along the x direction to determine the work:

$$\sum F_x = F_2\cos 60 - F_1 = -0.5N$$

$$W = F_{NET}d = (-0.5N)(-3.00m) = +1.5J$$

The net force in the x-direction is negative, which means that it points left. Since the displacement is also negative, the work done is positive, and the kinetic energy increases (the trunk's speed increases).

$$K_f - K_i = \Delta K = W = 1.5J$$

Example: Work and kinetic energy.

Chapter 7, problem 15. A firehose 12m long is uncoiled by pulling the nozzle end horizontally along a frictionless surface at a steady speed of 2.3 m/s. The mass of 1.0m of the hose is 0.25kg. How much work has been done on the hose by the applied forces when the entire hose is moving?

This problem certainly appears difficult. How do we calculate the force on the the firehose? Doesn't the amount of mass vary as the hose is being pulled out? Etc. It turns out that this problem is easy to solve if we use the work-kinetic energy theorem.

The hose is initially at rest, so the kinetic energy is:

$$K_i = \frac{1}{2} m v_i^2 = 0$$

When the hose is fully pulled out, it's entire mass (which =(0.25kg/1.0m)(12.0m)=3.0kg) is moving at the speed of 2.3m/s, so its final kinetic energy is:

$$K_f = \frac{1}{2} m v_f^2 = \frac{1}{2}(3.0\text{kg})(2.3\text{m/s})^2 = 7.9\text{J}$$

And therefore the work done is:

$$W = \Delta K = K_f - K_i = 7.9\text{J}$$