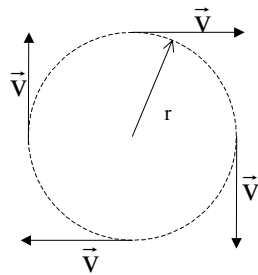


Uniform Circular Motion

Uniform circular Motion

- An object moving at constant speed in a circle
- The magnitude of the velocity remains constant
- The direction of the velocity changes continuously



Since acceleration is rate of change of velocity:

$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t}$$

The velocity changes if **either**:

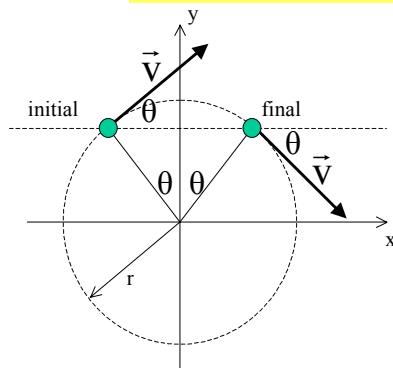
- its magnitude changes
- its direction changes

Lets try to determine the magnitude and direction of the acceleration, in terms of the constant magnitude of the velocity v , and the radius of the circle r .

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1

Acceleration in Uniform Circular Motion



Let's determine the average acceleration as the object (the green ball) moves

- from position marked "initial"
- to that marked "final".

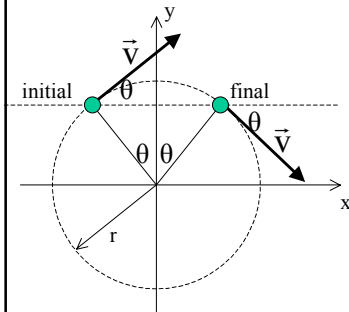
These two positions were chosen so that the line from the center of the circle to those positions make an angle θ with the y axis.

The other two angles marked θ come from the fact that the velocity vectors in uniform circular motion are **tangent to the circle** at all times.

Lectures 13, 14: Circular

2

Acceleration in Uniform Circular Motion



Distance the ball travels from initial to final time
= arc length of circle =

$$d = (2\theta)r$$

Since the ball travels at constant speed v , we can get the time from initial to final:

$$t = \frac{d}{v} = \frac{2\theta r}{v}$$

The components of the initial velocity:

$$v_{xi} = v \cos\theta \qquad v_{yi} = v \sin\theta$$

The components of the final velocity:

$$v_{xf} = v \cos\theta \qquad v_{yf} = -v \sin\theta$$

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3

Acceleration in Uniform Circular Motion

From the above information, we can get the average acceleration as the object moves from the initial to the final position:

$$a_{\text{avg},x} = \frac{\Delta v_x}{\Delta t} = 0 \qquad a_{\text{avg},y} = \frac{\Delta v_y}{\Delta t} = \frac{-v \sin\theta - v \sin\theta}{2r\theta/v} = \frac{-v^2 \sin\theta}{r\theta}$$

This is just the average acceleration. To get the instantaneous acceleration, we need to let the angle θ shrink to zero.

Using the fact that: $\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1$ $a_x = 0$ $a_y = -\frac{v^2}{r}$

We find that at the top of the circle, the instantaneous acceleration points downward - toward the center of the circle. In fact, if we pick any other point around the circle, we will find that the acceleration in uniform circular motion has the magnitude:

$$a = \frac{v^2}{r}$$

and points toward the center of the circle. This means that the acceleration vector is NOT constant!

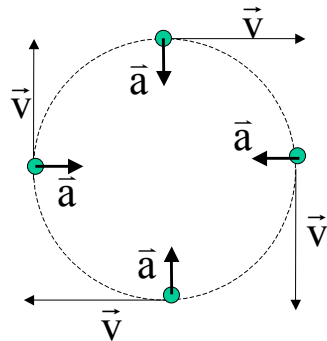
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Acceleration in Uniform Circular Motion

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\vec{a}

Lectures 13, 14: Circular

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Kinematics of Uniform Circular Motion

For an object moving at constant speed v in a circle of radius r :

Acceleration:

$$a = \frac{v^2}{r}$$

Since this acceleration points to the center of the circle, it is called centripetal (for “center-seeking”) acceleration.

Distance traveled in 1 revolution:

$$D = 2 \pi r$$

Time (or period) for 1 revolution:

$$T = \frac{2 \pi r}{v}$$

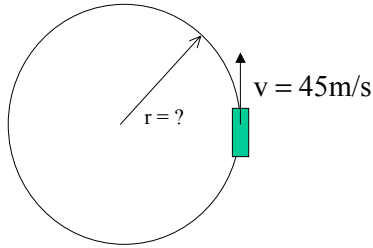
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Example: Centripetal Acceleration:

A car claims that its “lateral acceleration” is 8.9m/s^2 (this is $0.91g$'s). This represents the maximum acceleration that car can undergo sideways to its motion without skidding.

If the car is traveling at a constant speed of 100mi/h ($\sim 45\text{m/s}$), what is the minimum radius of curve it can negotiate without skidding?



$$r = \frac{v^2}{a} = \frac{(45)^2}{8.9} = 230\text{m}$$

Note:

- If the speed increases, the radius necessary increases by the square
- If the curve is banked, we will find the the minimum radius necessary is smaller
- What is providing the acceleration?

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Dynamics of Uniform Circular Motion

Newton's 1st Law: A body moves in constant speed in a straight line unless a net force acts on it to change its motion

If an object is moving in a circle, there must be a force acting on it.

Newton's 2nd Law: $\sum \vec{F} = m\vec{a}$ We usually think of this as
“A net force on an object of mass m causes an acceleration.”

We can turn this thinking around. In uniform circular motion, we determined that there is an acceleration. If there is an acceleration, there must be a force causing this acceleration.

If an object is undergoing uniform circular motion, it is being acted on by a net centripetal (“center-seeking”) force:

$$\sum F = ma = \frac{mv^2}{r}$$

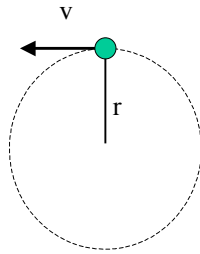
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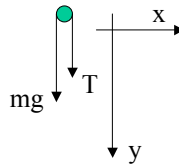
Example: Ball swinging in a vertical circle

A ball is being swung in a vertical circle. The mass of the ball is 150g, and the radius of the circle is 1.1m.

A) What is the minimum speed the ball must have at the top of the circle so that it continues to move in a circle?



At the top of the circle, there are two forces acting on the ball:

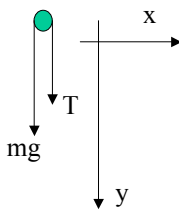


Note that we are defining downward to be positive
- really defining *inward* (toward the circle center) as positive.

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Example: Ball swinging in a vertical circle



$$\sum F = ma = \frac{mv^2}{r}$$

$$+ T + mg = \frac{mv^2}{r}$$

This last equation says that as the speed of the ball increases, T (the tension in the string) also must increase (since mg and r are constant).

This certainly conforms with our experience. But also note that as v gets smaller, T must decrease. Eventually, if v gets small enough, T becomes zero, and the string will no longer be under tension. The minimum speed occurs when T=0:

$$mg = \frac{mv_{\min}^2}{r}$$

$$v_{\min} = \sqrt{rg}$$

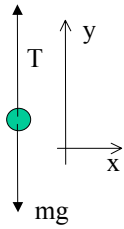
$$v_{\min} = 3.28\text{m/s}$$

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Example: Swinging ball continued

B) If the speed is actually twice the minimum determined in part A, what is the tension in the string at the *bottom* of the circle?



$$\sum F = ma = \frac{mv^2}{r}$$

$$+T - mg = \frac{mv^2}{r}$$

$$T = mg + \frac{mv^2}{r}$$

$$T = (0.15)(9.80) + \frac{(0.15)(6.56)^2}{1.1}$$

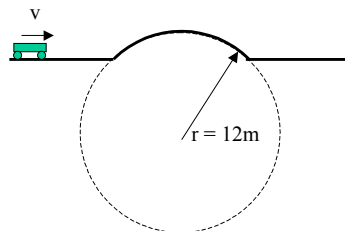
$$T = 7.34\text{N}$$

Note that we are now defining upward to be positive - really defining *inward* (toward the circle center) as positive.

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Example: Car going over a hump in the road

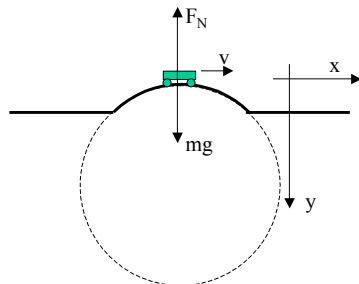


What is the minimum speed such that the car will leave the road at the top of the hump?

$$\sum F = ma = \frac{mv^2}{r}$$

$$-F_N + mg = \frac{mv^2}{r}$$

As v increases, F_N must decrease.



Minimum v occurs when F_N becomes zero.

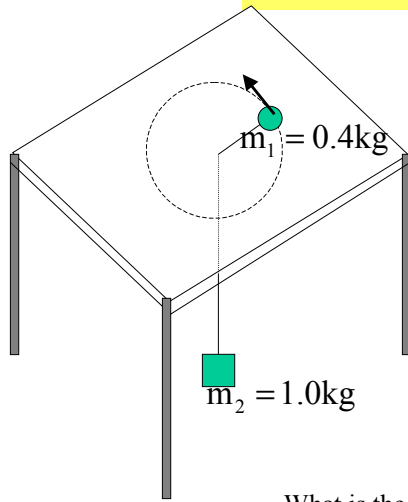
$$mg = \frac{mv_{\min}^2}{r}$$

$$v_{\min} = \sqrt{rg}$$

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Example: Puck and weight



A puck is sliding on a frictionless table. It is connected to a string that goes through a hole in the center of the table. The other end of the string is attached to a weight.

What is the speed of the puck necessary so that the puck moves in a circle with a radius of 0.8m (and such that the weight m_2 remains stationary)?

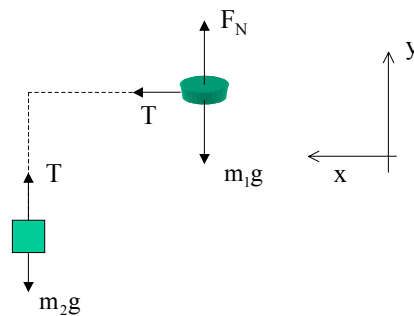
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Example: Puck and weight

SIDE VIEW:

Free body diagram for puck



Free body diagram for weight

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Example: Puck and weight (cont)

Apply Newton's 2nd Law to the puck (inward is positive):

$$\sum F_x = m_1 a_x = \frac{m_1 v^2}{r}$$
$$+ T = \frac{m_1 v^2}{r}$$
$$v = \sqrt{\frac{rT}{m_1}}$$

What is T?

Apply Newton's 2nd Law to the weight:

$$\sum F_y = m_2 a_y = 0$$
$$+ T - m_2 g = 0$$
$$T = m_2 g$$

Now we can solve for the speed:

$$v = \sqrt{\frac{rT}{m_1}} = \sqrt{\frac{r g m_2}{m_1}}$$
$$v = \sqrt{\frac{(0.8\text{m})(9.80\text{m/s}^2)(1.0\text{kg})}{0.4\text{kg}}}$$
$$v = 4.4 \text{ m/s}$$

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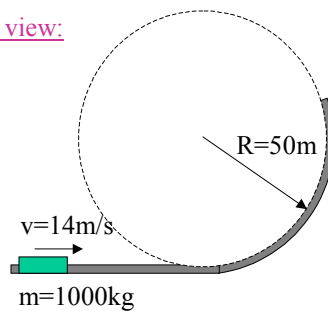
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Example: Car on Curve: Flat Road

A car approaches a curve with a speed of 14m/s (about 50km/h). The radius of curvature of the road is 50m.

Will the car the car skid? Assume that for a dry road the coefficient of static friction is 0.6, while for a wet road it is 0.25.

Top view:



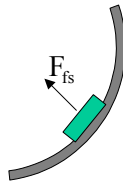
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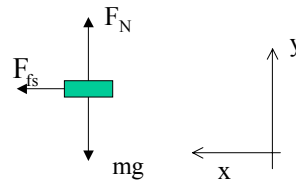
Example: Car on Curve: Flat Road

What is the centripetal force that keeps the car on the road and moving in a circle? It is the **force of friction** pushing the tires from the side. It is **static friction - as long as** the tires do not skid.

View from above the car:



View from behind the car:



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Example (cont): Car on Curve: Flat Road

Determine the maximum allowed static friction for the two road conditions:

A) Dry road $F_{fs}^{\max} = \mu_s F_N = (0.6)(9800) = 5900\text{N}$

Car will stay on the road. Note that the friction force is 3900N, and that it will increase if R is smaller or v is bigger.

B) Wet road $F_{fs}^{\max} = \mu_s F_N = (0.25)(9800) = 2500\text{N}$

Car will skid off of the road.

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Example (cont): Car on Curve: Flat Road

Apply Newton's 2nd Law to the car:

$$\sum F_x = ma_x = \frac{mv^2}{r}$$

$$+ F_{fs} = \frac{mv^2}{r}$$

$$F_{fs} = \frac{(1000\text{kg})(14\text{m/s})^2}{50\text{m}}$$

$$F_{fs} = 3900\text{N}$$

$$\sum F_y = ma_y = 0$$

$$+ F_N - mg = 0$$

$$F_N = mg = (1000\text{kg})(9.80\text{m/s}^2)$$

$$F_N = 9800\text{N}$$

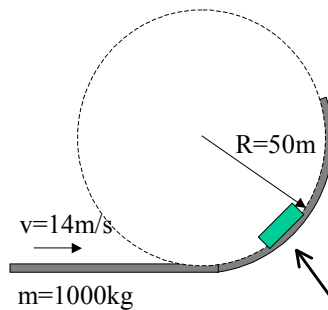
What this means is that 3900N is needed to keep the car turning and on the road. Is this bigger than the maximum allowed static friction?

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Example: Car on Curve: Banked Road

View from above the car:



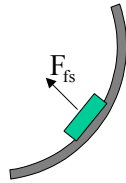
Curves like this are usually "banked".
Why?

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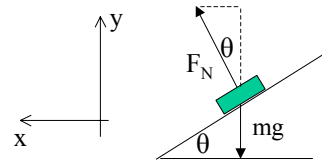
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Example: Car on Curve: Banked Road

View from above the car:



View from behind the car:



θ is the angle that the road is “banked” at. Can this be chosen so that we don’t need to rely on friction at all? Yes! In the figure above, the x -component of the normal force can provide the force pushing the car towards the center of the circle.