

# Friction

When an object slides over a surface, there is usually some **resistance to this sliding**. This is due to a frictional force, and is always directed opposite the intended direction of motion along the surface. The size of this force is dependent on the particular object and surface.

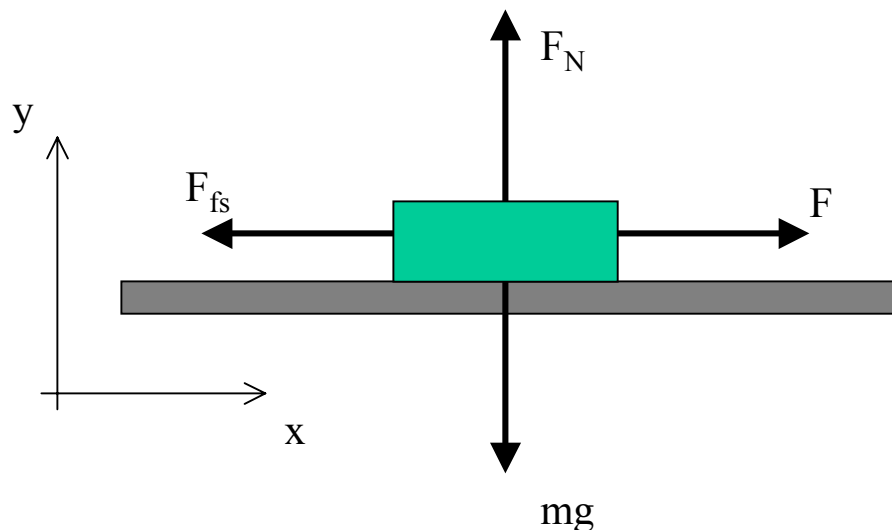
Imagine a block on a rough horizontal surface. A force  $F$  is applied to the block, pulling it to the right. Imagine that this force is very slowly increased from zero to some larger value.

We know from experience that when the force is very small, the block does not move. This can only occur if there is some other force which is directed opposite to the applied force. This is the force of friction.

Since the block is not moving, this force is called *static friction*.

# Friction

A picture of this situation would be:

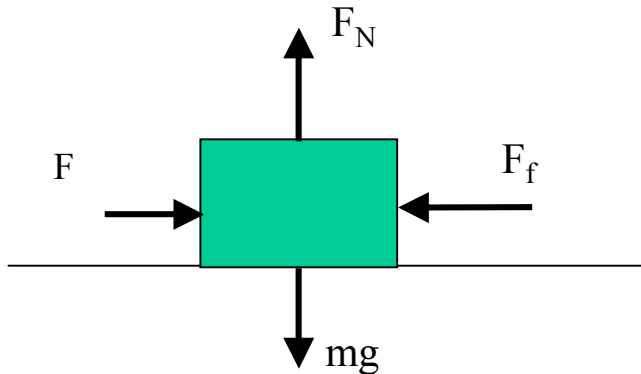


Note:

- 1)  $F$  is the applied force, and it is (in this case) directed along the x-axis.
- 2)  $F_{fs}$  is the force of static friction. It is directed opposite the intended motion. Since the block would move along the  $+x$  direction if the friction were not present, it is directed along the  $-x$  axis.
- 3) The force of friction will always be parallel to the surface.

# Static Friction

Let's apply Newton's 2nd Law to this problem (consider the motion along the x-axis):



$$\begin{aligned}\sum F_x &= ma_x = 0 \\ F - F_{fs} &= 0 \\ F_{fs} &= F\end{aligned}$$

Note:

- 1)  $a_x$  equals zero because the block is stationary.
- 2) As  $F$  gets larger, so does  $F_{fs}$ .

We know that eventually, when  $F$  gets large enough, the block will move. When  $F$  is just large enough, so that the block is *about* to move (but is not moving yet), the force of static friction will have a maximum value. Let's call this

$$F_{fs}^{\max}$$

# Static Friction

Experimentally, we find that:  $F_{fs}^{\max} = \mu_s F_N$

Note:

- 1)  $\mu_s$  is called the coefficient of static friction. Its value is dependent upon the two surfaces in question.
- 2) The maximum static friction force is dependent on the size of the normal force.
- 3) The maximum static friction force is not dependent on the on the area of contact.

If the block is not on the verge of moving:  $F_{fs}^{\max} < \mu_s F_N$

## Kinetic Friction

If the applied force is large enough, the block will move. As the block moves, there is still a frictional force present. It is still in the direction opposite to the motion, and it is still parallel to the surface of contact between the block and the rough surface.

Experimentally, we find that:

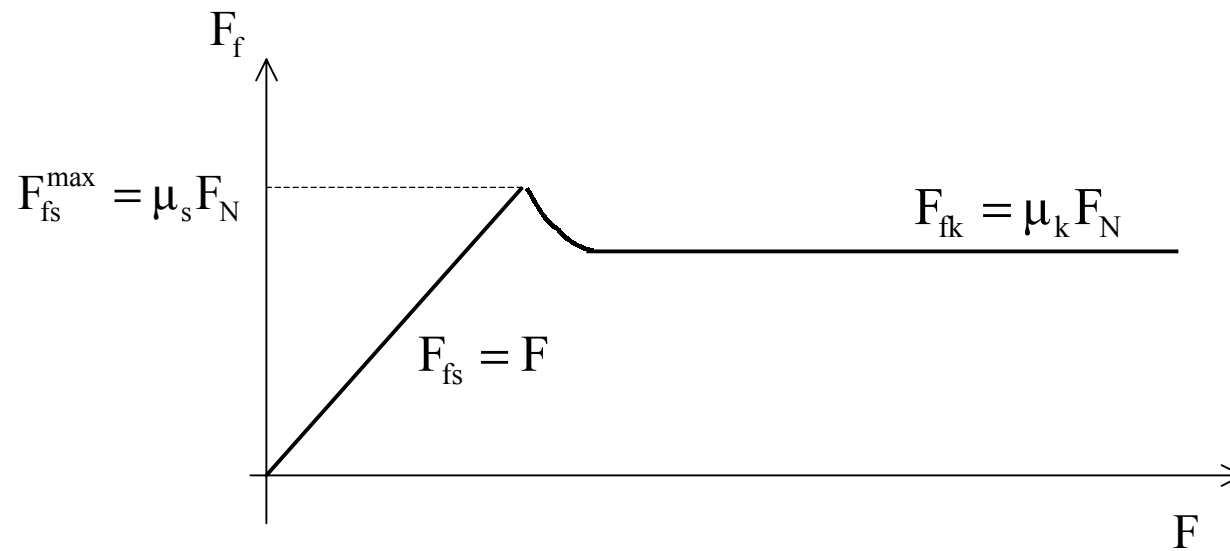
$$F_{fk} = \mu_k F_N$$

Note:

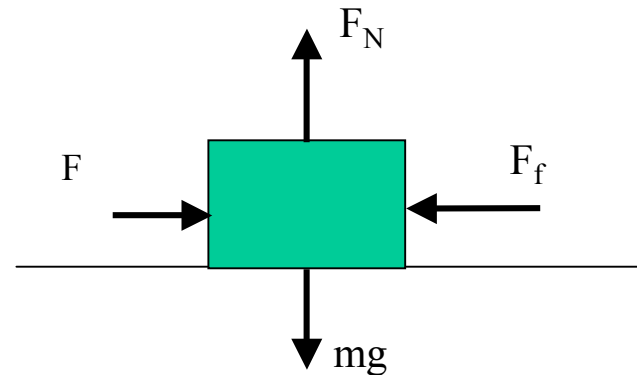
- 1)  $\mu_k$  is called the coefficient of kinetic friction.  
Its value is dependent upon the two surfaces in question.
- 2)  $\mu_k$  is independent of how fast the object is moving across the surface.
- 3)  $\mu_k < \mu_s$  (this means that the force of kinetic friction is *less than* the maximum force of static friction)

# Static and Kinetic Friction

Plot of frictional force vs applied force  $F$ .



## Friction summary



Static Friction, with applied force small

$$F_{fs} < \mu_s F_N$$

Static Friction, with applied force just large enough to get object to start moving

$$F_{fs}^{\max} = \mu_s F_N$$

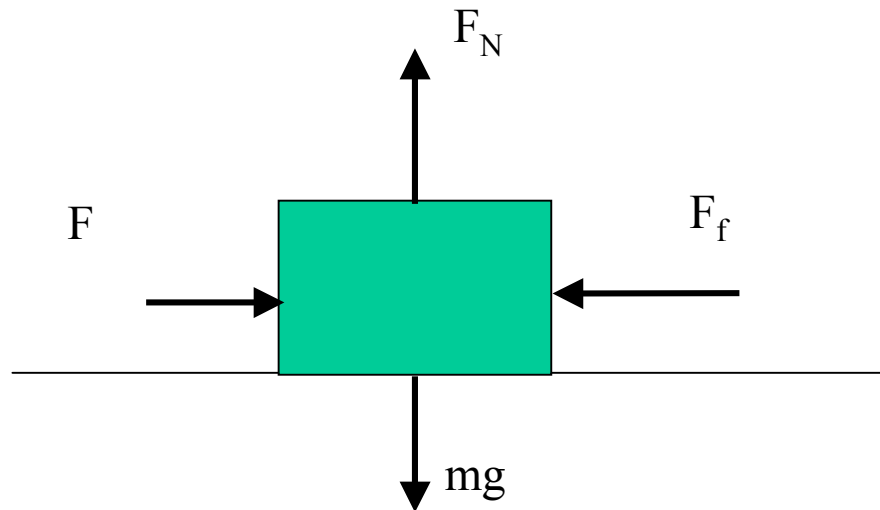
Kinetic Friction, with applied force and the object moving

$$F_{fk} = \mu_k F_N$$

Example: A wooden crate with mass 100kg is at rest on a stone floor. You know that the coefficients of kinetic and static friction are:  $\mu_s=0.5$  and  $\mu_k=0.4$ .

A) What is the minimum horizontal force  $F$  need to just get the crate moving?

I) Draw a picture of the problem:



Example: Wooden crate continued.

II) Apply Newton's 2nd Law:

$$\begin{aligned}\sum F_x &= ma_x = 0 \\ F - F_{fs} &= 0 \\ F &= F_{fs}\end{aligned}$$

$$\begin{aligned}\sum F_y &= ma_y = 0 \\ F_N - mg &= 0 \\ F_N &= mg\end{aligned}$$

III) Use the relationship between Friction and the Normal force:  $F_{fs} = \mu_s F_N$

$$F = F_{fs} = \mu_s F_N = \mu_s mg$$

$$F = (0.5)(100\text{kg})(9.80\text{m/s}^2)$$

$$F = 490\text{N}$$

Conclusion: The crate will move only if a force  $> 490\text{N}$  is applied to it.

Example (Continued):

B) Now say that the crate is moving. What is the minimum force needed to keep the crate in motion at constant velocity?

I) Apply Newton's 2nd Law:

$$\begin{aligned} \sum F_x = ma_x = 0 & \quad \text{Constant velocity} & \quad \sum F_y = ma_y = 0 \\ F - F_{fk} = 0 & & \quad F_N - mg = 0 \\ F = F_{fk} & & \quad F_N = mg \end{aligned}$$

III) Use the relationship between Friction and the Normal force:  $F_{fk} = \mu_k F_N$

$$\begin{aligned} F &= F_{fk} = \mu_k F_N = \mu_k mg \\ F &= (0.4)(100\text{kg})(9.80\text{m/s}^2) \\ F &= 392\text{N} \end{aligned}$$

The crate will move only if a force  $> 490\text{N}$  is applied to it, but after it starts moving, only  $392\text{N}$  is needed to keep it moving at constant velocity.

Example (continued):

C) What happens if a horizontal force of 500N is applied?

Quick:  $F > 490\text{N}$  so crate will start moving.  $F > 392\text{N}$  so after crate starts moving, it accelerates.

What is the acceleration of the block?

I) Apply Newton's 2nd Law:

$$\begin{aligned} \sum F_x &= ma_x & \sum F_y &= ma_y = 0 \\ F - F_{fk} &= ma_x & F_N - mg &= 0 \\ a_x &= \frac{F - F_{fk}}{m} & F_N &= mg \end{aligned}$$

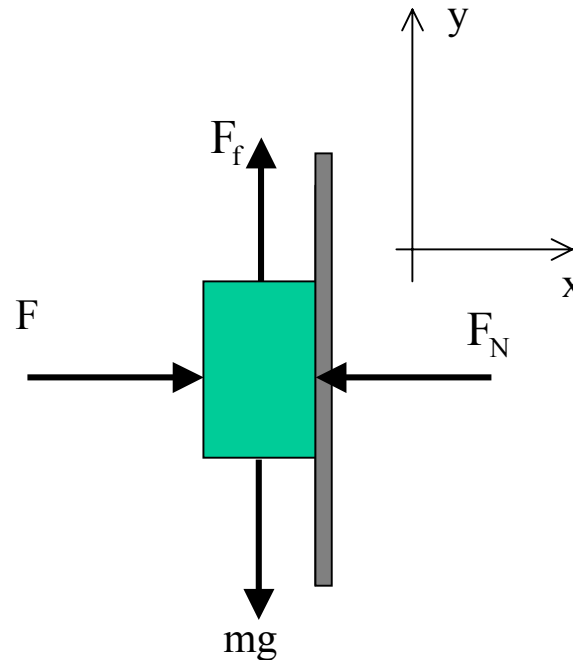
II) Use the relationship between Friction and the Normal force:  $F_{fk} = \mu_k F_N$

$$\begin{aligned} a_x &= \frac{F - F_{fk}}{m} = \frac{F - \mu_k F_N}{m} = \frac{F - \mu_k mg}{m} \\ a_x &= \frac{500\text{N} - (0.4)(100\text{kg})(9.80\text{m/s}^2)}{100\text{kg}} \\ a_x &= 1.08\text{m/s}^2 \end{aligned}$$

Example (Ch 6 prob xx): A horizontal force  $F$  of 12N pushes a block which weighs 5.0N against a vertical wall. The coefficient of static friction between the wall and the block is 0.60, and the coefficient of kinetic friction is 0.40. Assume that the block is not moving initially.

A) Will the block start moving?

I) Draw a picture of the problem:



## Example (Ch 6 prob xx): Continued

II) Apply Newton's 2nd Law:

$$\begin{aligned}\sum F_x &= ma_x = 0 \\ F - F_N &= 0 \\ F &= F_N\end{aligned}$$

$$\begin{aligned}\sum F_y &= ma_y \\ F_f - mg &= ma_y\end{aligned}$$

What is  $a_y$ ?

We are not told that the acceleration of the block in the y-direction is zero - only that the block is not moving initially. Will the block start to move? If so  $a_y$  is not zero!

## Example (Ch 6 prob xx): Continued

What is the maximum force of static friction?

$$F_{fs}^{\max} = \mu_s F_N = \mu_s F = (0.6)(12\text{N}) = 7.2\text{N}$$

What is the weight of the block?

$$mg = 5.0\text{N} \quad mg < F_{fs}^{\max}$$

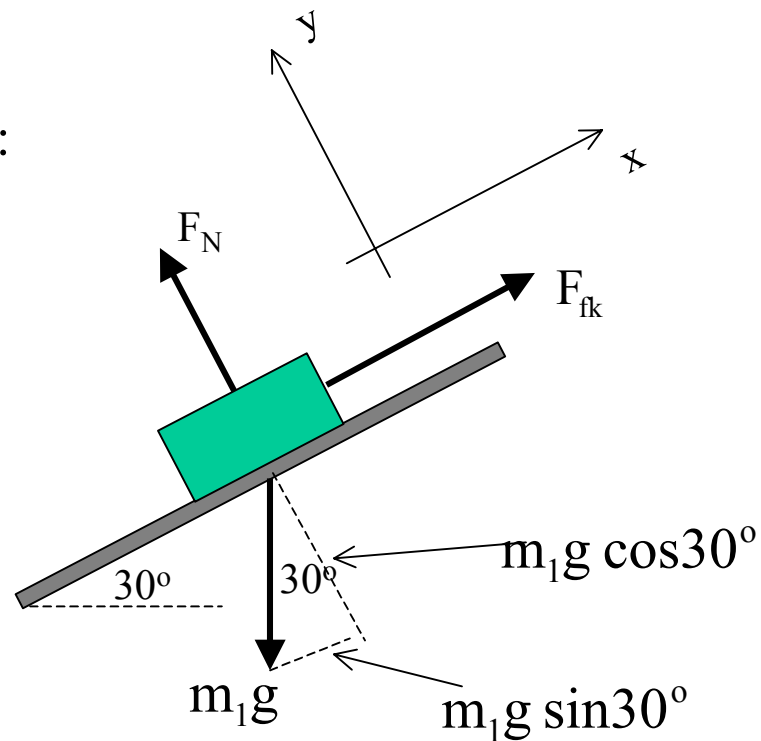
Remember, we were told that the block was not moving initially. Since the weight of the block is not enough to start the block moving, it remains at rest.

What is the actual force of friction? You should be able to determine this very quickly!

Not as quick: If the value of  $F$  is lower slowly, at what  $F$  will the block start to move?

Example: A block is placed on an incline plane. It is given a nudge so that it starts moving down the plane. If the coefficient of kinetic friction between the block and the plane is 0.20, what is the block's acceleration down the plane?

I) Draw a picture of the problem:



Example: Block on an incline plane.

II) Apply Newton's 2nd Law:

$$\sum F_x = ma_x$$

$$F_{fk} - mg \sin\theta = ma_x$$

$$\sum F_y = ma_y = 0$$

$$F_N - mg \cos\theta = 0$$

$$F_N = mg \cos\theta$$

III) Use the relationship between Friction and the Normal force:  $F_{fk} = \mu_k F_N$

$$F_{fk} - mg \sin\theta = ma_x$$

$$\mu_k F_N - mg \sin\theta = ma_x$$

$$\mu_k mg \cos\theta - mg \sin\theta = ma_x$$

$$\mu_k g \cos\theta - g \sin\theta = a_x$$

NOTE: The acceleration is independent of the mass of the block!

for  $\theta = 30^\circ$  :

$$a_x = (0.2)(9.80)(0.87) - (9.80)(0.5)$$

$$a_x = -3.20\text{m/s}^2$$