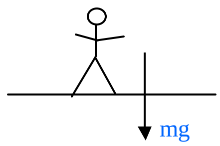
Gravitation

We have already talked about the force of gravity:



Where:

m is the mass of the person

g is a constant

The main assumption needed to use the above expression for the force of gravity is that the person needed to be very close to the surface of the earth.

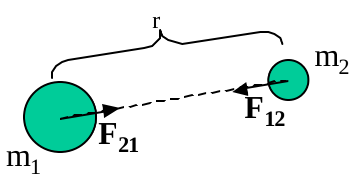
However, the force of gravity acts beyond just the surface of the earth.
The force of gravity is responsible for:

- Keeping the moon in orbit around the earth
- Keeping the earth in orbit around the sun
- Keeping the solar system as part of the galaxy
- etc

Gravity acts between all things with mass.

Newton's Law of Universal Gravitation

The gravitational force between any two bodies of mass m_1 and m_2, separated by a distance r, is given by:



$$F_G = \frac{Gm_1m_2}{r^2}$$

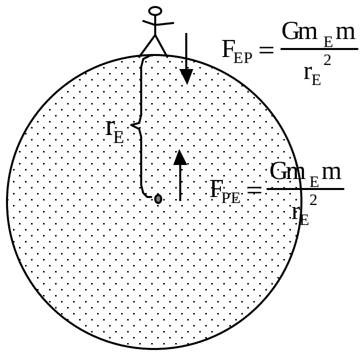
$$G = 6.67 \times 10^{-11}\ N \cdot m^2 / kg^2$$

Note:

1) Object m_1 attracts object m_2 with force $\mathbf{F_{12}}$
2) This force is attractive, and is directed towards m_1
3) Object m_2 attracts object m_1 with force $\mathbf{F_{21}}$
4) This force is also attractive and directed towards m_2

$$|F_{12}| = |F_{21}| = F_G$$

Gravity Near the Surface of the Earth



F_{EP} = Force of earth on the person

F_{PE} = Force of person on the earth

$$|F_{EP}| = |F_{PE}| = F_G$$

If the person always stays very close to the surface of the earth, then r_E can be approximated as a constant.

The force of gravity (due to the earth) on an object with mass m very near the earth's surface:

$$F_G = \frac{Gm_E m}{r_E^2} = \left(\frac{Gm_E}{r_E^2}\right) m = mg \qquad \left(\frac{Gm_E}{r_E^2}\right) \equiv a_g = 9.83 \text{m/s}^2$$

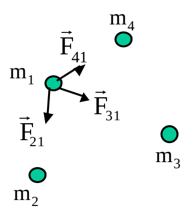
Interesting things about Gravity

$$F_G = \frac{Gm_1 m_2}{r^2}$$

- G is a measure of the "strength" of gravity. It is very very small.
- So the force of gravity will be large only when at least one of the two masses in the above expression are large.
- But the force of gravity has unlimited range. F becomes zero only in the limit that r goes to infinity.
- This last statement implies that you are currently exerting a (small) force on galaxies far, far away.
- The above expression is only strictly true for "pointlike" particles. How is it that we can use it for "extended" objects like the earth, a car, ourselves…?

Principle of Superposition

Principle of superposition: The net effect is the sum of individual effects.



The total gravitational force on particle m_1 due to particles m_2, m_3, and m_4, is the vector sum of the gravitational forces:

$$\vec{F}_1 = \vec{F}_{21} + \vec{F}_{31} + \vec{F}_{41}$$

Or more generally for "n" particles:

$$\vec{F}_1 = \sum_{i=2}^{n} \vec{F}_{i1}$$

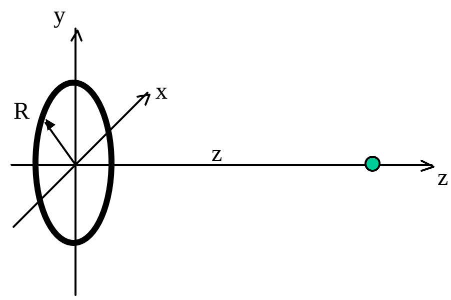
If we want to know the force that a real object (made up of a continuous distribution of particles) exerts on another particle of mass m, we need to replace the above discrete sum by a continuous integral:

$$F_1 = \int dF = \int \frac{Gm_1}{r^2} dm$$

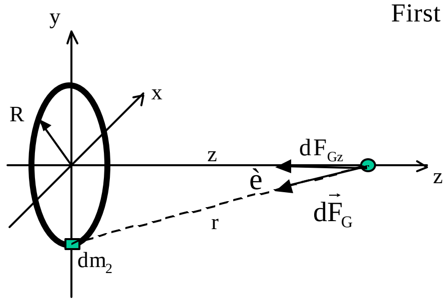
Example: Force on a Particle due to a Ring:

A particle of mass m_1 is located a distance z away from a thin ring of mass m_2, along its axis. The radius of the ring is R.

Determine the magnitude and direction of the gravitational force of the ring, on the particle m_1.



Example: Force on a Particle due to a Ring:



First consider a small element dm_2 of the ring.

dF_{Gz} : This is the z-component of the force of gravity due to mass element dm_2.

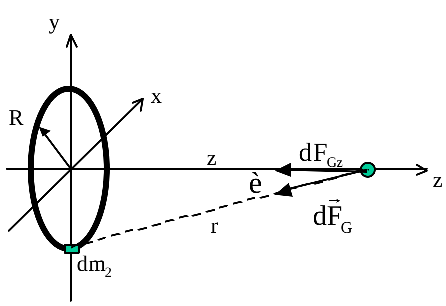
Notice that the component of the force of gravity ***perpendicular*** to this will be canceled by a mass element on the opposite side of the ring.

The z-component of the force of gravity exerted on mass m_1 by mass element dm_2 is:

$$dF_{Gz} = \left(\frac{Gm_1 dm_2}{r^2}\right)\cos\theta$$

Example: Force on a Particle due to a Ring:

First consider a small element dm_2 of the ring.



Since: $r^2 = z^2 + R^2$

and $\cos\theta = \frac{z}{r}$

then

$$dF_{Gz} = \frac{Gm_1 dm_2}{r^2}\cos\theta = \frac{z\,Gm_1 dm_2}{\left(z^2 + R^2\right)^{3/2}}$$

The force of gravity exerted on mass m_1 by the whole ring is then:

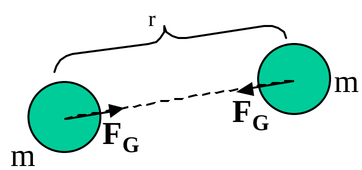
$$F_{Gz} = \int dF_{Gz} = \frac{z\,Gm_1 m_2}{\left(z^2 + R^2\right)^{3/2}}$$

Ex: Force on mass due to a thin Ring

NOTE:

1) If the mass m_1 is located at z=0, the gravitational force goes to zero.
2) Newton showed using similar methods that a uniform spherical shell of mass m attracts particles outside of the shell as if all of the shell's mass were located at its center.
   The same can be demonstrated for a uniform sphere.
3) If a particle is located anywhere inside a uniform shell, the net gravitational attraction on it is zero.

Example: Gravitational Attraction between Two Spheres



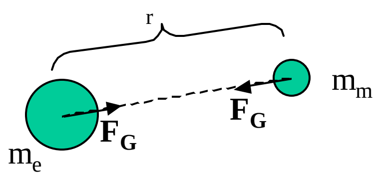
Both spheres have mass m=100kg, and are separated by a distance of 1.00m.

$$F_G = \frac{Gm_1 m_2}{r^2} = \frac{(6.67\times 10^{-11})(100)(100)}{(1.00)^2}$$

$$F_G = 6.67\times 10^{-7}\,\text{N}$$

This is small. About the weight of a baby flea. Will not pull a lineman offsides.

Example: Gravitational Attraction between The Earth and the Moon



$m_m = 7.35\times10^{22}$ kg
$m_e = 5.98\times10^{24}$ kg
$r = 384\times10^{3}$ m

$$F_G = \frac{Gm_1m_2}{r^2} = \frac{(6.67\times10^{-11})(7.35\times10^{22})(5.98\times10^{24})}{(384\times10^{3})^2}$$

$$F_G = 1.99\times10^{20}\,\mathrm{N}$$

This is big. Equivalent to the thrust of 6 million Saturn 5 moon rockets.