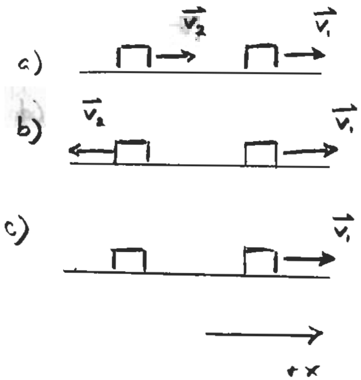
Q9.



No external forces are acting on the exploding box. It separates because of its own internal energy. So, because of conservation of momentum, the total linear momentum of the system (here, the exploding box) cannot change.

$\vec{P}$ = constant

Arrows on vectors can be dropped because everything occurs in the x-direction.

m_1 = mass of piece 1, m_2 = mass of piece 2

For each of the three cases, the system has total momentum $P_i = (m_1+m_2)V$, where V = box's velocity before explosion.

For case a, $m_1v_1 + m_2v_2 = (m_1+m_2)V$; $P_f = m_1v_1 + m_2v_2$

$$\Rightarrow v_1 = \frac{1}{m_1}\left((m_1+m_2)V - m_2v_2\right)$$

case b, $m_1v_1 + m_2(-v_2) = (m_1+m_2)V$

$$\Rightarrow v_1 = \frac{1}{m_1}\left((m_1+m_2)V + m_2v_2\right)$$

case c, $m_1v_1 + m_2(0) = (m_1+m_2)V$

$$\Rightarrow v_1 = \frac{1}{m_1}(m_1+m_2)V$$

So now it can be seen that:

v_1 for case a < v_1 for case c, and

v_1 for case b > v_1 for case c.

So the magnitudes of $\vec{v}_1$ for each cases are ranked greatest to smallest as: b, c, a

Problems:

1.

a) $$x_{com} = \frac{m_E x_E + m_m x_m}{m_E + m_m}$$ where E subscript means "Earth"

m subscript means "moon"

$$= \frac{(5.98\times10^{24})(0) + (7.36\times10^{22})(3.82\times10^{8})}{5.98\times10^{24} + 7.36\times10^{22}}$$

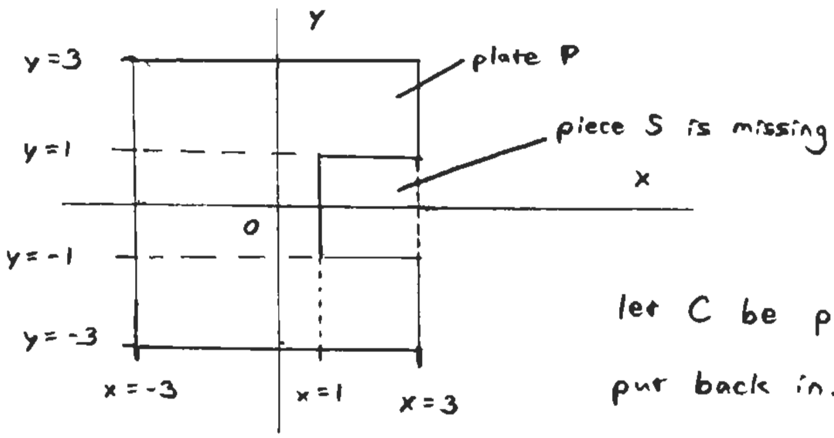
$$= 4.64\times10^{6}\ m$$

b) Earth's radius $= 6.37\times10^{6}\ m = R_E$

So the fraction is $\dfrac{4.64\times10^{6}}{6.37\times10^{6}} = 0.728$

or $x_{com} = 0.728\,R_E$

5.



let C be plate P with missing piece S put back in.

Know that $x_{com_C} = 0$ since plate C is centered at the origin and it is uniform.

P and S can be treated as point masses at their centers of mass.

Also the expression for $x_{com_C} = x_{com_{S+P}} = \dfrac{m_S x_{com_S} + m_P x_{com_P}}{m_S + m_P}$

so $\dfrac{m_S x_{com_S} + m_P x_{com_P}}{m_S + m_P} = 0 \Rightarrow m_S x_{com_S} + m_P x_{com_P} = 0$

Therefore $x_{com_P} = -\dfrac{m_S}{m_P} x_{com_S}$

Know that $x_{com_S} = 2$ by symmetry. Still need $\dfrac{m_S}{m_P}$.

$$\frac{m_S}{m_P} = \frac{\text{density}_S \times \text{thickness}_S \times \text{area}_S}{\text{density}_P \times \text{thickness}_P \times \text{area}_P}$$

But the plate is uniform, so $\text{density}_S = \text{density}_P$, and $\text{thickness}_S = \text{thickness}_P$.

$$\Rightarrow \frac{m_S}{m_P} = \frac{\text{density}_S \times \text{thickness}_S \times \text{area}_S}{\text{density}_S \times \text{thickness}_S \times \text{area}_P} = \frac{\text{area}_S}{\text{area}_P}$$

From the picture, $\text{area}_S = 2 \times 2 = 4\ m^2$

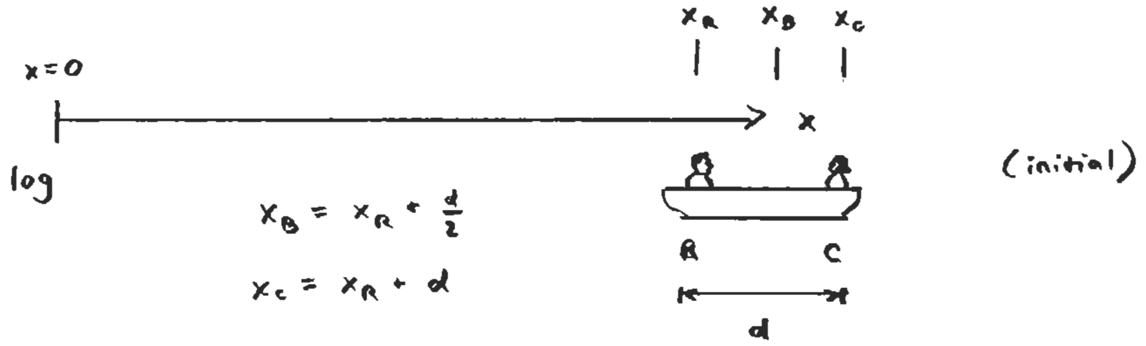
$\text{area}_P = (6 \times 6) - (2 \times 2) = 32\ m^2$

so $x_{com_P} = -\dfrac{m_S}{m_P} x_{com_S} = -\dfrac{\text{area}_S}{\text{area}_P} x_{com_S} = -\dfrac{4}{32}(2) = -\dfrac{1}{4}$

For the y-direction, the above method could be used, but the plate is symmetric about the x-axis, so it can just be seen that $y_{com_3} = 0$.

So the coordinates of the center of mass of the remaining piece are $(-\frac{1}{4}, 0)$.

18.



$x_B = x_R + \frac{d}{2}$

$x_C = x_R + d$

Conservation of momentum means that the coordinate of the center of mass must be the same before & after the people switch places.

before: $$x_{com} = \frac{m_R x_R + m_B(x_R + \frac{d}{2}) + m_C(x_R + d)}{m_R + m_B + m_C}$$

after: $$x_{com} = \frac{m_R(x_R - \Delta x + d) + m_B(x_R - \Delta x + \frac{d}{2}) + m_C(x_R - \Delta x)}{m_R + m_B + m_C}$$

where Δx = the magnitude of the change in the boat's x-coordinate = 0.40 m

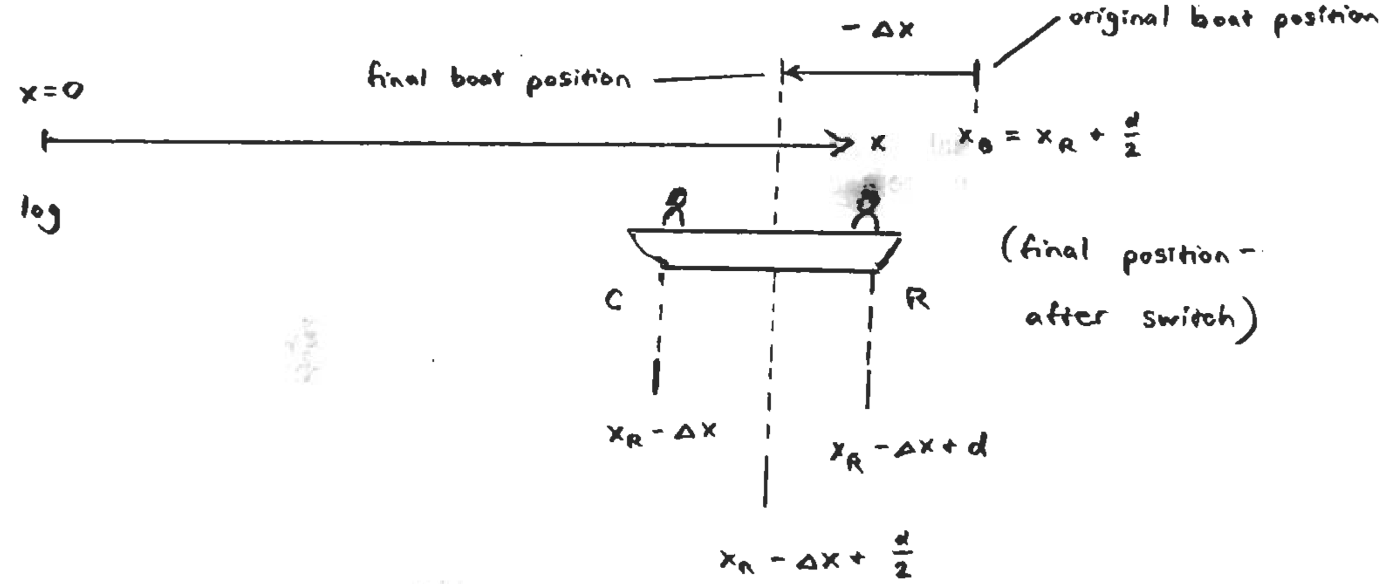
So equating x_{com}'s expressions for before and after the switch:

(denominators are the same, so they cancel)

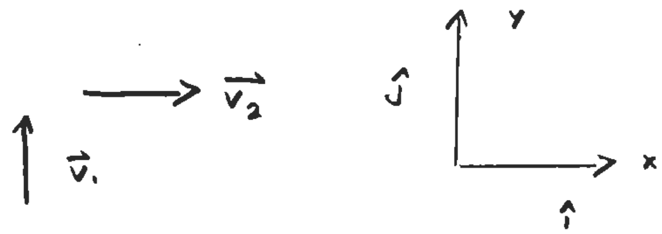
$$m_R x_R + m_B(x_R + \frac{d}{2}) + m_C(x_R + d) = m_R(x_R - \Delta x + d) + m_B(x_R - \Delta x + \frac{d}{2}) + m_C(x_R - \Delta x)$$

Solving for m_C: $$m_C = \frac{-\Delta x(m_B + m_R) + d\,m_R}{d + \Delta x}$$

So $m_c = \dfrac{-0.40(30+80) + 3.0(80)}{3.0 + 0.40} = 58 \text{ kg}$



23.



a) change in the truck's kinetic energy:

$41 \frac{km}{h} = 11.39 \frac{m}{s}$

$51 \frac{km}{h} = 14.17 \frac{m}{s}$

so $K_1 = \frac{1}{2}(2100)(11.39^2) = 1.36 \times 10^5 \text{ J}$

$K_2 = \frac{1}{2}(2100)(14.17^2) = 2.11 \times 10^5 \text{ J}$

$\Rightarrow \Delta K = K_2 - K_1 = 2.11 \times 10^5 - 1.36 \times 10^5 = 7.5 \times 10^4 \text{ J}$

b) magnitude and direction of the change in the truck's linear momentum:

$\vec{v_1} = 11.39\hat{j}$

$\vec{v_2} = 14.17\hat{i}$

$\Delta\vec{p} = \vec{p_2} - \vec{p_1}$ and $\vec{p_1} = m\vec{v_1}$, $\vec{p_2} = m\vec{v_2}$

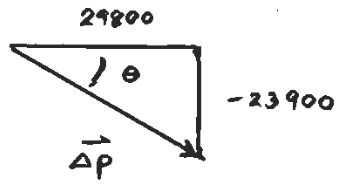
so $\Delta\vec{p} = (2100)(14.17)\hat{i} - (2100)(11.39)\hat{j}$

$= 29800\hat{i} - 23900\hat{j}$

magnitude: $|\Delta\vec{p}| = ((29800)^2 + (-23900)^2)^{\frac{1}{2}}$

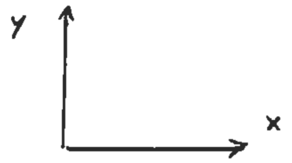
$= 3.82 \times 10^4 \text{ J}$

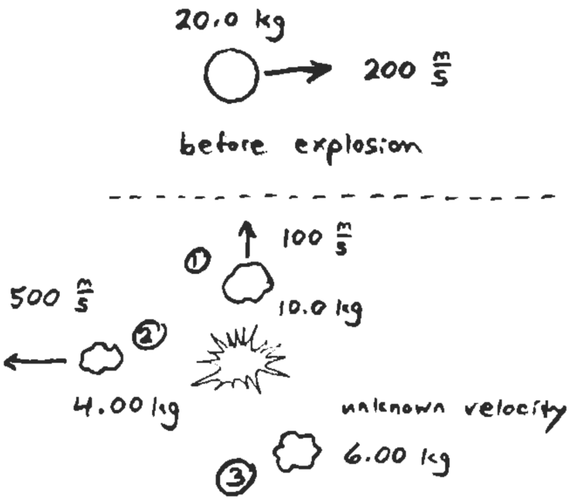
c) direction:



$\tan\theta = \frac{23900}{29800} \Rightarrow \theta = 38°$ south of east

37.





The explosion is the result of internal forces of the system. So the net external force on the system is zero, so momentum is conserved.

a) $\vec{P_i} = M\vec{V_i}$ = total momentum before collision

$= (20.0)(200)\,\hat{i} = 4000\,\hat{i}$

$\vec{P_f}$ = total momentum after collision

$\vec{P_i} = \vec{P_f}$ must be true

$\vec{P_f} = m_1\vec{v_{1f}} + m_2\vec{v_{2f}} + m_3\vec{v_{3f}}$

$= (10.0)(100)\,\hat{j} + (4.00)(-500)\,\hat{i} + (6.00)\,\vec{v_{3f}}$

$= 1000\,\hat{j} - 2000\,\hat{i} + 6.00\,\vec{v_{3f}}$ and $\vec{v_{3f}} = v_{3f_x}\,\hat{i} + v_{3f_y}\,\hat{j}$

$\vec{P_i} = \vec{P_f}$

$4000\,\hat{i} = 1000\,\hat{j} - 2000\,\hat{i} + 6.00\left(v_{3f_x}\,\hat{i} + v_{3f_y}\,\hat{j}\right)$

Do the x- and y- directions separately:

x: $4000 = -2000 + 6.00\,v_{3f_x}$

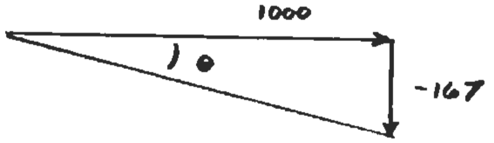
$\Rightarrow v_{3f_x} = 1000 \ \frac{m}{s}$

y: $0 = 1000 + 6.00\,v_{3f_y}$

$\Rightarrow v_{3f_y} = -167 \ \frac{m}{s}$

The speed of the third fragment is $v = \sqrt{1000^2 + (-167)^2} = 1010 \ \frac{m}{s}$

$\vec{v}_{3f} = 1000\hat{i} - 167\hat{j}$



$\tan\theta = \dfrac{167}{1000}$

$\Rightarrow \theta = 9.5°$

b) The energy released is just the change in the system's kinetic energy.

$K_i = \frac{1}{2} M V_i^2 = \frac{1}{2}(20.0)(200^2) = 4 \times 10^5 \text{ J}$

$K_f = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2 + \frac{1}{2} m_3 v_{3f}^2$. Factor out the $\frac{1}{2}$.

$= \frac{1}{2}\left((10.0)(100)^2 + (4.00)(500)^2 + (6.00)(1010)^2\right)$

$= 3.56 \times 10^6 \text{ J}$

So $\Delta K = K_f - K_i = 3.56 \times 10^6 - 4 \times 10^5 = 3.2 \times 10^6 \text{ J}$

40. $\Delta K = K_f - K_i = 16 \text{ J}$ $\longrightarrow +x$

before: M, 8.0 kg, $V_i = 2.0 \frac{m}{s}$

after: m_1, 4.0 kg; m_2, 4.0 kg

$$K_i = \tfrac{1}{2} M V_i^2$$

$$K_f = \tfrac{1}{2} m_1 v_{1f}^2 + \tfrac{1}{2} m_2 v_{2f}^2$$

$$K_i = \tfrac{1}{2}(8.0)(2.0)^2 = 16 \text{ J}$$

$$K_f = \tfrac{1}{2}(4.0) v_{1f}^2 + \tfrac{1}{2}(4.0) v_{2f}^2$$

$$\Delta K = 16 = 2.0 v_{1f}^2 + 2.0 v_{2f}^2 - 16 \quad (= K_f - K_i)$$

$$\Rightarrow 16 = v_{1f}^2 + v_{2f}^2$$

No external forces act on the system, so momentum is conserved.

$$P_i = P_f \qquad P_i = MV_i, \quad P_f = m_1 v_{1f} + m_2 v_{2f}$$

$$P_i = (8.0)(2.0) = 16 \text{ kg} \cdot \frac{m}{s}$$

$$P_f = (4.0) v_{1f} + (4.0) v_{2f}$$

$$\Rightarrow 16 = 4.0 v_{1f} + 4.0 v_{2f}$$

$$4 = v_{1f} + v_{2f}$$

$$\Rightarrow v_{1f} = 4 - v_{2f}$$

Now use this in the ΔK expression:

$$16 = v_{1f}^2 + v_{2f}^2$$

$$16 = (4 - v_{2f})^2 + v_{2f}^2$$

$$16 = 16 - 8v_{2f} + v_{2f}^2 + v_{2f}^2$$

$$0 = 2v_{2f}^2 - 8v_{2f}$$

$$v_{2f}^2 - 4v_{2f} = 0$$

$$v_{2f}(v_{2f} - 4) = 0$$

$$\Rightarrow v_{2f} = 0 \quad \text{or} \quad v_{2f} = 4$$

If $v_{2f} = 0$, $v_{1f} = 4 - v_{2f} = 4 - 0 = 4 \frac{m}{s}$

If $v_{2f} = 4$, $v_{1f} = 4 - 4 = 0 \frac{m}{s}$