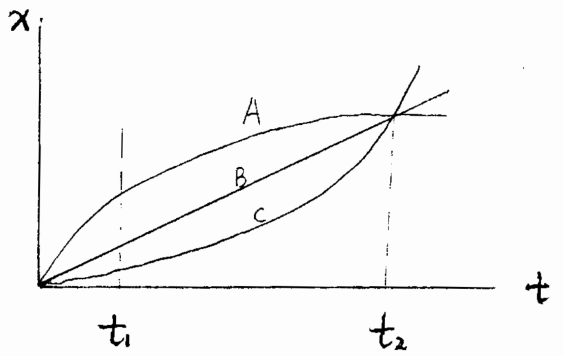
Physics 131 Homework 6

Ch 7 Question 5



(a)

Recall the kinetic energy $K = \frac{1}{2} m V^2$

the velocity $V = \frac{dx}{dt}$ (the slope of the x-t curve)

Thus, at time t_1

$V_A > V_B > V_C$ (speed)

and $K_A > K_B > K_C$ (kinetic energy)

(b) At time t_2, the same criterion with (a), one gets

$V_C > V_B > V_A$

and $K_C > K_B > K_A$

(c) Recall the Work-Kinetic Energy Theorem:

$W = \Delta K$

Look at each x-t curve, one may find that

	Velocity at time t_1		Velocity at time t_2	Change of kinetic energy ΔK
A	V_{A1}	$>$	V_{A2}	$\Delta K_A < 0$
B	V_{B1}	$=$	V_{B2}	$\Delta K_B = 0$
C	V_{C1}	$<$	V_{C2}	$\Delta K_C > 0$

Therefore, the net work done on the three boxes should be

$$W_C > W_B > W_A$$

(d) The sign of net work tells us how the energy is transfered.

A: $W_A < 0$ —— (2)

B: $W_B = 0$ —— (3)

C: $W_C > 0$ —— (1)

Ch 7 Question 8

If one ~~use~~ use "W_F" to denote the work done by "your force", "W_G" to denote the work done by "the gravitational force of the armadillo". According to the Work - Kinetic Energy theorem.

$$W_F + W_G = 0$$ (here we assume the armadillo is in rest at the beginning and the end of the process)

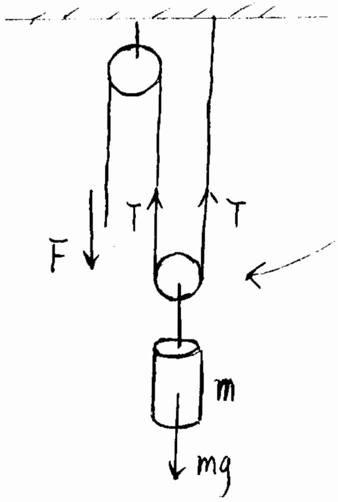
where $W_G = mg \cdot h \cos(180°) = -mgh$ (h is the height)

Thus $W_F = mgh$. From here, one can conclude that the work done by "your force" W_F

depends on (1) (2) (3)

doesn't depend on (d) (e)

Ch 7 Problem 15



(a) Can think about the force diagram of the free massless pulley.

When speed is constant, the forces on the pulley should be in balance, according to Newton's 2rd law,

$$2T - mg = 0$$

$\therefore$ the tension on the cord $T = \frac{1}{2}mg$

$\therefore$ the force on the free end of the cord

$$F = T = \frac{1}{2}mg = \frac{1}{2} \times 20 \times 9.8 = 98\ N$$

(b) If the canister is lifted by 2.0 cm, two segments of the cord at the two sides of the free pulley should shorten by the same amount. Thus, the distance pulled at the free end of the cord should be $2.0\ cm \times 2 = 4.0\ cm = 0.04\ m$

(c) The work done by "your force"

$$W_F = \vec{F} \cdot \vec{d} = F d \cos(0°) = Fd = 98 \times 0.04 = 3.92\ N$$

(d) The work done by the gravitational force on the canister

$$W_G = mg \cdot d' \cos(180°) = -mgd' = -20 \times 9.8 \times 0.02 = -3.92\ N$$

where d — the distance moved at the free end of the cord

d' — the distance moved by the canister

Ch 7 Problem 22

(a) Here, we can make a unit conversion first.

$$k = 2.5\ N/cm = \frac{2.5N}{0.01\ m} = 250\ N/m$$

$$d = 12\ cm = 0.12\ m \qquad m = 250g = 0.25\ kg$$

The work done by the gravitational force is

$$W_G = mgd\cos(0^\circ) = mgd = 0.25 \times 9.8 \times 0.12 = 0.29\ J$$

(b) The work done by the spring force is

$$W_F = \int_0^{0.12} kx\cos(180^\circ)\,dx = -\int_0^{0.12} kx\,dx = -\frac{1}{2}kx^2\Big|_0^{0.12}$$

$$= -\frac{1}{2} \times 250 \times 0.12^2 = -1.8\ J$$

(c) Denote "V" as the speed of the block just before it hits the spring. One can apply the Work - Kinetic Energy theorem to the block during the process from its just hitting the spring to "momentarily stopping"

$$W_{net} = \Delta K$$

$$W_F + W_G = 0 - \frac{1}{2}mV^2$$

$$\therefore -1.8 + 0.29 = -\frac{1}{2} \times 0.25 \times V^2$$

$$\therefore V = 3.48\ m/s$$

(d) If the speed is doubled $V' = 2 \times 3.48 = 6.96\ m/s$

Now, denote d' as the maximum distance of the compression of spring. The analoges of W_F, W_G are

$$W_F' = -\frac{1}{2} k d'^2$$

$$W_G' = mgd'$$

Still use Work – Kinetic Energy theorem

$$W_F' + W_G' = 0 - \frac{1}{2} m V'^2$$

$$\therefore\ mgd' - \frac{1}{2} k d'^2 = -\frac{1}{2} m V'^2$$

$$\therefore\ 0.25 \times 9.8 \times d' - \frac{1}{2} \times 250 \times d'^2 = -\frac{1}{2} \times 0.25 \times 6.96^2$$

$$\therefore\ 125 d'^2 - 2.45 d' - 6.05 = 0$$

One only needs the positive solution, since $d' > 0$

$$d' = \frac{2.45 + \sqrt{2.45^2 + 4 \times 125 \times 6.05}}{2 \times 125} = 0.23\ m$$

Ch 7 Problem 35

Denote W_G as the work done by the gravitational force of the total mas.

W_m as the work done by the motor

W_c as the work done by the couterweight

Apply the Work - Kinetic Energy theorem,

$$W_G + W_m + W_c = \Delta K = 0$$

$$\therefore W_m = -W_G - W_c$$

And $W_G = -m_t g d = -1200 \times 9.8 \times 54 = -635040 \text{ J}$

$W_c = m_c g d = 950 \times 9.8 \times 54 = 502740 \text{ J}$

$$\therefore W_m = 635040 - 502740 = 132300 \text{ J}$$

$\therefore$ the average power of the motor is

$$P_m = \frac{W_m}{t} = \frac{132300}{3 \times 60} = 735 \text{ W}$$

Ch 8 Question 9

For region "AB" and "BC", one can consider the conservation of mechanical energy. For region "CD", the frictional force will do work and cost the total mechanical energy.

	External Force (friction)	Total Mechanical Energy	Gravitational Potential Energy	Kinectic Energy
AB	no	constant	decreases	increases
BC	no	constant	increases	decreases
CD	non-zero	decreases	constant	decreases

Ch 8. Problem 6

(a) From P to Q

$W_G = mg\,(5R - R) = mg \cdot 4R = 4mgR$

(b) From P to the top of the loop

$W_G' = mg\,(5R - 2R) = mg \cdot 3R = 3mgR$

(c) At P.

$U = mg \cdot (5R - 0) = 5mgR$

(d) At Q

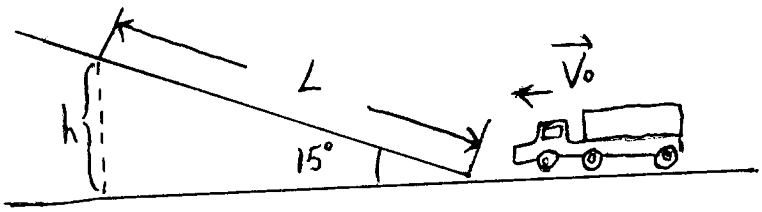
$U = mg\,(R - 0) = mgR$

(e) At the top of the loop,

$U = mg\,(2R - 0) = 2mgR$

(f) If one looks at the formula to calculate the work done by the gravitational force and the ~~energy~~ potential energy, they don't depend on the speed. Thus, the result of (a) through (e) remain the same.

Ch 8. Problem 13



(a) If denote "h" as the height achieved by the truck when it momentarily stops on the ramp, the mechanical energy is conserved since the ramp is frictionless.

$$\therefore \quad \frac{1}{2} m V^2 = m g h$$

$$\therefore \quad h = \frac{V^2}{2g}$$

The minimum length "L" has the relation with "h"

$$\frac{h}{L} = \sin 15°$$

$$\therefore \quad L = \frac{h}{\sin 15°} = \frac{V^2}{2g \cdot \sin 15°} \simeq \frac{36.11^2}{2 \times 9.8 \times \sin 15°} \simeq 257 \text{ m}$$

(notice $V = 130 \text{ km/h} = 36.11 \text{ m/s}$)

(b) Since $L = \dfrac{V^2}{2g \sin 15°}$ does not depend on the truck's mass, "L" remains the same with the decrease of mass.

(C) The "L" will decrease as the speed "v" decreases

Ch 8 Problem 19

(a) The elastic potential energy

$U = \frac{1}{2} k x^2 = \frac{1}{2} \times 1960 \times 0.2^2 = 39.2\ J$

(note $k = 19.6\ N/cm = \frac{19.6N}{0.01m} = 1960\ N/m$)

(b) Conside the conservation of mechanical energy.

$\Delta K + \Delta U_E + \Delta U_G = 0$

ΔK —— change of kinetic energy

ΔU_E —— change of elastic potential energy

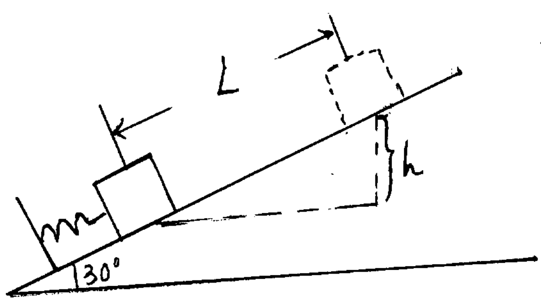
ΔU_G —— change of gravitational potential energy

$\because$ $\Delta K = 0$

$\Delta U_E = 0 - 39.2 = -39.2\ J$

$\therefore$ $\Delta U_G = -\Delta K - \Delta U_E = 39.2\ J$

(c)



If "h" is the height achieved by the block.

$\Delta U_G = mgh = 39.2\ J$

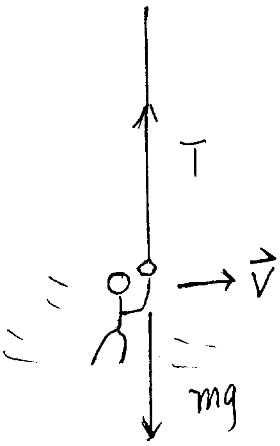
$\therefore$ $h = \frac{39.2}{mg} = \frac{39.2}{2 \times 9.8} = 2\ m$

$\because$ $L \sin 30° = h$

$\therefore$ $L = \frac{h}{\sin 30°} = \frac{2}{0.5} = 4\ m$

Ch 8 Problem 26

(a) Suppose Tarzan could move to the bottom, check the force on the vine.



At the bottom, the speed is "V". consider the conservation of mechanical energy.

$$mgh = \frac{1}{2} m V^2$$

$$\therefore \quad mV^2 = 2mgh$$

Look at the force diagram at this moment.

$$T - mg = \frac{mV^2}{r}$$

$$\therefore \quad T = mg + \frac{mV^2}{r} = mg + \frac{2mgh}{r}$$

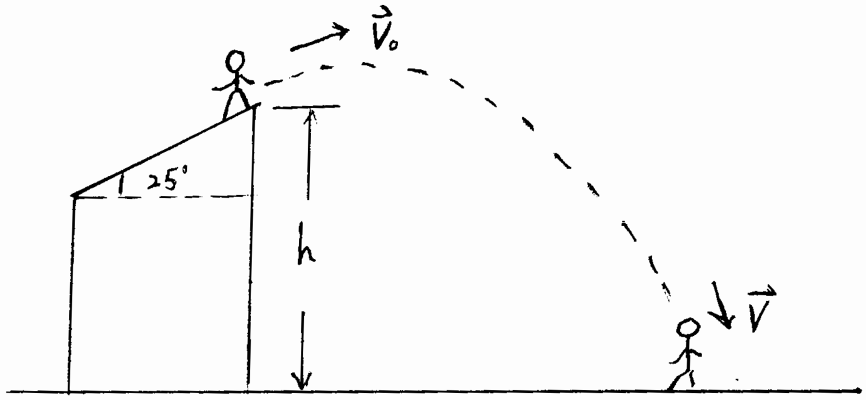
$$= mg \cdot \left(1 + 2\frac{h}{r}\right) = 688 \times \left(1 + 2 \times \frac{3.2}{18}\right)$$

$$\approx 932.63\ N$$

$\therefore \quad T < 950\,N$, the vine will not break

(b) Tarzan will have largest speed at bottom, since he has the maximum kinetic energy at this moment. And the force achieves the greatest value at this time, i.e 932.63 N.

Ch 8. Problem 45



If the gravitational potential of the skier at the groung is taken to be zero.

The mechanical energy at the beginning is

$$mgh + \frac{1}{2}mV_0^2 = 60 \times 14 \times 9.8 + \frac{1}{2} \times 60 \times 24^2 = 25512\,J$$

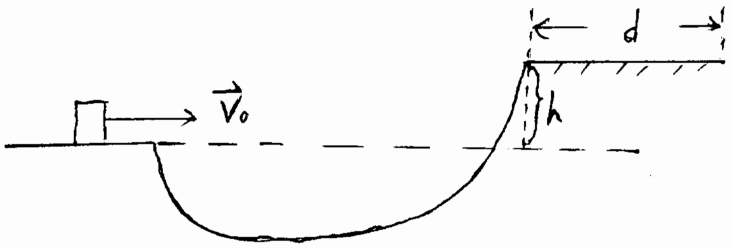
The mechanical energy at the end is

$$\frac{1}{2}mV^2 = \frac{1}{2} \times 60 \times 22^2 = 14520\,J$$

$\therefore$ the reduced mechanical energy is

$$25512 - 14520 = 10992\,J$$

Ch 8 Problem 59



The frictional force should be $f = mg\mu$.

The work done by the frictional force is equal to the change of mechanical energy

$$\Delta E_{mec} = \frac{1}{2} m V_0^2 - mgh$$

$$\therefore f \cdot d = mg\mu \cdot d = \frac{1}{2} m V_0^2 - mgh$$

$$\therefore d = \frac{\frac{1}{2} V_0^2 - gh}{g\mu} = \frac{V_0^2}{2g\mu} - \frac{h}{\mu} = \frac{6^2}{2 \times 9.8 \times 0.6} - \frac{1.1}{0.6}$$

$$= 1.2\,m$$