Ch 10, problem 33

$$E_i = E_f \quad \text{(no friction)}$$

Conservation of energy: $K_i + U_i = K_f + U_f$

$$\underbrace{\tfrac{1}{2} m_1 v_1^2 + \tfrac{1}{2} m_2 v_2^2}_{K_i} = \underbrace{\tfrac{1}{2}(m_1+m_2) v^2}_{K_f} + \underbrace{\tfrac{1}{2} k X^2}_{U_f} \quad (1)$$

and $U_i = 0$ because at the beginning the spring is uncompressed.

X = maximum compression of the spring

from (1)

$$X = \frac{1}{\sqrt{k}} \sqrt{m_1 v_1^2 + m_2 v_2^2 - (m_1+m_2) v^2} \quad (2)$$

find v using conservation of total momentum

$$p_i^{TOT} = p_f^{TOT}$$

$$m_1 v_1 + m_2 v_2 = (m_1+m_2) v \Rightarrow v = \frac{m_1 v_1 + m_2 v_2}{m_1+m_2} \quad (3)$$

plug (3) into (2)

$$X = \frac{1}{\sqrt{k}} \sqrt{m_1 v_1^2 + m_2 v_2^2 - (m_1+m_2)\left(\frac{m_1 v_1 + m_2 v_2}{m_1+m_2}\right)^2}$$

plug in values given by the problem

$$\boxed{X = 0.25\ m}$$

Ch 10, problem 40

v_{1i} = velocity of ball right before collision

v_{1f} = - - - - after -

v_{2i} = - - block - before -

v_{2f} = - - - - after -

- find v_{1i} by using conservation of energy

$$\frac{1}{2} m_1 v_{1i}^2 = m_1 g h ,$$

where $h \equiv$ hight of the ball before it was released = 70.0cm

$$v_{1i} = \sqrt{2gh} = \sqrt{2(9.8\tfrac{m}{s^2})(0.7m)} = 3.7\tfrac{m}{s}$$

- Use equation (10-30) in book, valid for elastic collisions in one dimension, when one of the objects is initially at rest

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} \qquad (10\text{-}30)$$

$$v_{1f} = \frac{0.5\,kg - 2.5\,kg}{0.5\,kg + 2.5\,kg} (3.7 m/s)$$

$$\boxed{v_{1f} = -2.47\ m/s} \Rightarrow \boxed{\text{speed} = |v_{1f}| = 2.47\tfrac{m}{s}}$$

- Use equation (10-31) to find v_{2f}.

$$v_{2f} = \frac{2 m_1}{m_1 + m_2} v_{1i}$$

$$\boxed{v_{2f} = 1.23 m/s} \Rightarrow \boxed{\text{speed} = |v_{2f}| = 1.23 m/s}$$

Ch 10, problem 41

(a)

- Use book's equation (10-31)

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i}$$

- Use problem statement $v_{1f} = \frac{v_{1i}}{4}$

$$\Rightarrow \quad v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} 4v_{1f}$$

$$m_1 + m_2 = 4m_1 - 4m_2$$

$$-3m_1 = -5m_2$$

$$m_2 = \frac{3}{5} m_1$$

$$\boxed{m_2 = 1.2 \text{ kg}}$$

(b)

$$v_{cm} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} \quad (1)$$

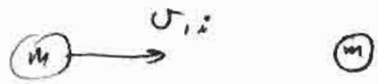
For a closed, isolated system, the velocity of the center of mass is not affected by collisions; so we can use equation (1) with the values of the velocities before the collision

$$v_{cm} = \frac{(2 \text{ kg})(4 \text{ m/s})}{(2 \text{ kg} + 1.2 \text{ kg})}$$

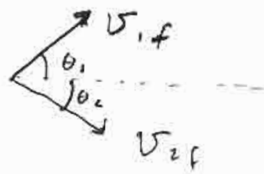
$$\boxed{v_{cm} = 2.5 \text{ m/s}}$$

Ch 10, problem 52

before collision



after collision



conservation of momentum in x-direction

$$\cancel{m} v_{1i} = \cancel{m} v_{1f} \cos\theta_1 + \cancel{m} v_{2f} \cos\theta_2 \quad (1)$$

conservation of momentum in y-direction

$$0 = v_{1f} \sin\theta_1 - v_{2f} \sin\theta_2 \quad (2)$$

(a)

(2) $\Rightarrow$ $$v_{2f} = \frac{v_{1f} \sin\theta_1}{\sin\theta_2} \quad (3)$$

(3) into (1)

$$v_{1i} = v_{1f} \cos\theta_1 + \frac{v_{1f} \sin\theta_1}{\sin\theta_2} \cos\theta_2$$

solve for θ_2

$$\theta_2 = \arctan\left(\frac{v_{1f} \sin\theta_1}{v_{1i} - v_{1f} \cos\theta_1} \right)$$

$$\boxed{\theta_2 = +\cancel{60}^\circ}\ 30^\circ$$

using (3) $$\boxed{v_{2f} = 1.9\ \mathrm{m/s}}$$

magnitude and direction of the ve of the other ball [see figure above]

continues...

Ch 10 problem 52 continued

(b)

$$E_i \stackrel{?}{=} E_f$$

$$\frac{1}{\cancel{2}}\cancel{m}\, v_{1i}^2 \stackrel{?}{=} \frac{1}{\cancel{2}}\cancel{m}\, v_{1f}^2 + \frac{1}{\cancel{2}}\cancel{m}\, v_{2f}^2$$

$$v_{1i}^2 \stackrel{?}{=} v_{1f}^2 + v_{2f}^2$$

$$(2.2)^2 \stackrel{?}{=} (1.1)^2 + \underbrace{(1.905255\ldots)^2}_{\text{use all digits obtained}}$$

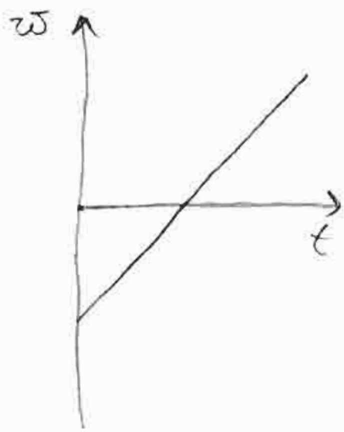
we verify in this way that energy was conserved.

$$\Rightarrow \boxed{\text{elastic collision}}$$

Week 8 HW Chpt 11

Chpt 11 Question 2

(a) The initial direction of rotation is clockwise. (−)



(b) The final direction of rotation is counter-clockwise (+).

(c) The disk momentarily stops since there is a time when $\omega = 0$

(d) The angular acceleration is the slope of the ω vs. t line, which is positive

(e) The slope of the line is constant ⇒ the angular acceleration is constant.

Chpt 11 Question 9

$$\sum \tau = I\alpha,$$

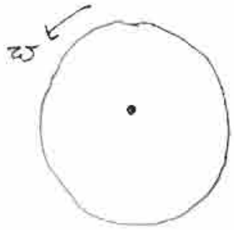
$$\tau = \vec{F}\cdot\vec{r} = |\vec{F}||\vec{r}|\sin\phi$$

The greatest angular acceleration is going to be max when torque is maximum. Torque is max when $\sin\phi$ is max.

$\sin\phi$ is greatest when $\phi = 90^\circ$ and the same for $\phi = 70^\circ$ and 110°.

Week 8 HW Chpt 11

Chpt 11 - #14



$\theta_0 = 0$

$\theta_f = 90$ rad

$\omega_0 = ?$

$\omega_f = ?$

$\alpha = 2.0 \text{ rad/s}^2$

$\Delta t = 3$ s

(a) How long was the wheel turning before the 3s interval?

Find expression for ω_0

$$\theta_f = \theta_0 + \omega_0(\Delta t) + \frac{1}{2}\alpha(\Delta t)^2$$

$$\omega_0 = \frac{\theta_f}{(\Delta t)} - \frac{1}{2}\alpha(\Delta t)$$

In terms of the starting time t_s

$$\omega_0 = \cancel{\omega_s}_{0} + \alpha t_s$$

$$\omega_0 = \alpha t_s = \frac{\theta_f}{(\Delta t)} - \frac{1}{2}\alpha(\Delta t)$$

$$\boxed{t_s = \frac{\theta_f}{\alpha \Delta t} - \frac{1}{2}\Delta t = \qquad t_s = \frac{90}{(2)(3)} - \frac{3}{2} = 13.5\text{s}}$$

(b) We found an expression for ω_0 already in part a.

$$\omega_0 = \frac{\theta_f}{(\Delta t)} - \frac{1}{2}\alpha(\Delta t)$$

$$\boxed{\omega_0 = \frac{90}{3} - \frac{1}{2}(2)(3) = 27 \text{ rad/s}}$$

Week 8 HW Chpt 11

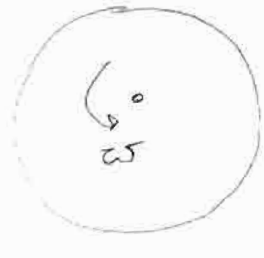
Chpt 11 #17

$\theta_0 = 0$

$\theta_f = 40 \text{ rev}$

$\omega_0 = 1.5 \text{ rad/s}$

$\omega_f = 0$



(a) $\omega_0 = 1.5 \text{ rad/s} = +0.239 \text{ rev/s}$

$$\underbrace{\omega_f^2}_{=0} = \omega_0^2 + 2\alpha\Delta\theta$$

$$\alpha = \frac{-\omega_0^2}{2(\Delta\theta)} = -7.12\times10^{-4} \text{ rev/s} = -4.5\times10^{-3} \text{ rad/s}$$

$$\underbrace{\omega_f}_{=0} = \omega_0 + \alpha t$$

$$\boxed{t = \frac{\omega_f}{\alpha} = 335 \text{ sec to come to rest}}$$

(b) The angular acceleration is what we found in (a)

$$\boxed{\alpha = \frac{-\omega_0^2}{2(\Delta\theta)} = -7.12\times10^{-4} \text{ rev/s} = -4.5\times10^{-3} \text{ rad/s}}$$

(c) The time to complete 20 revolutions is

$$\theta_f = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\Rightarrow t = \frac{-\omega_0 \pm \sqrt{\omega_0^2 + 2\theta\alpha}}{\alpha} = \frac{-0.239 \pm \sqrt{0.239^2 + 2(20)(-7.12\times10^{-12})}}{-7.12\times10^{-12}}$$

Week 8 HW Chpt 11

Chpt 11 #17

The eqn has two positive roots

$$t = 98\,s \quad \text{and} \quad t = 572\,s$$

It should take longer for 40 revolutions than for 20. We throw out $t = 572$ since it is greater than 335.

$$\boxed{t = 98\,s}$$

Chpt 11 #28

$r = 2.83\ cm$

$\alpha = 14.2\ rad/s^2$

$\omega_0 = 0$

$\omega_f = 2760\ rev/min \left(\frac{2\pi\ rad}{1\ rev}\right)\left(\frac{1\ min}{60\ sec}\right) = 289\ rad/sec$

(a) The tangential acceleration is

$$\boxed{a = \alpha r = (14.2\ rad/s^2)(2.83\ cm) = 40.2\ cm/s^2}$$

(b) The radial acceleration at full speed is

$$\boxed{a_r = \omega^2 r = (289)^2 (0.0283\ m) = 2.36 \times 10^3\ m/s^2}$$

(c) The angular distance covered by a point on the rim is

$$\omega_f^2 = \cancel{\omega_0^2}^{\,0} + 2\alpha\Delta\theta$$

$$\Rightarrow \boxed{\Delta\theta = \frac{\omega_f^2}{2\alpha} = \frac{(289)^2}{2(14.2)} = 2.94 \times 10^3\ rad}$$

Week 8 HW Chpt 11

Chpt 11 #28

ⓒ The linear displacement is

$$S = r\theta = (0.0283)(2.94 \times 10^3) = 83.2\,m$$

Chpt 11 #31

ⓐ The angular velocity of the pulsar is

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi}{T}$$

So, the angular acceleration is

$$\alpha = \frac{d\omega}{dt} = -\frac{2\pi}{T^2}\frac{dT}{dt}$$, and from the problem,

$$\frac{dT}{dt} = \frac{1.26 \times 10^{-5}\, s/y}{3.16 \times 10^{7}\, s/y} = 4.00 \times 10^{-13}$$

Therefore, $$\alpha = -\left(\frac{2\pi}{(0.0335)^2}\right)(4.00 \times 10^{-13}) = -2.3 \times 10^{-9}\,\frac{rad}{s^2}$$

ⓑ To find the time it will take to stop

$$\omega_f = \omega_0 + \alpha t \qquad \text{where } \omega_f = 0,\ \omega_0 = \frac{2\pi}{T}$$

$$-\frac{\omega_0}{\alpha} = t \Rightarrow \frac{-2\pi}{(-2.3 \times 10^{-9})(0.33)} = 8.3 \times 10^{10}\,s$$

$$\frac{8.3 \times 10^{10}\,s}{} \cdot \frac{1\,y}{3.16 \times 10^{7}\,s} \approx 2600 \text{ years}$$

Week 8 HW Chpt 11

chpt 11 #31

(c) The initial speed of the pulsar was

$$\omega_{now} = \omega_0 + \alpha t$$

The time is

$$(2005 - 1054) \text{ years} \frac{3.16 \times 10^7 \text{ s}}{1 \text{ year}} = 3 \times 10^{10} \text{ s}$$

So

$$\omega_{now} - \alpha t = \omega_0$$

$$\boxed{\omega_0 = \left(\frac{2\pi}{T}\right) - \left(-2.3 \times 10^{-9} \text{ rad/s}^2\right)\left(3 \times 10^{10} \text{ s}\right) = 259 \text{ rad/s}}$$

It's period was

$$\boxed{T = \frac{2\pi}{\omega} = \frac{2\pi}{259} = .024 \text{ s}}$$