Hint for homework Set No. 3, Problem 2.11

Deadline – Wednesday, Oct. 14, 2009

In part b) you are asked for an analytic expression (in polar coordinates) that gives the electrostatic potential Φ at any point outside the cylinder, and to derive its asymptotic form far from the cylinder.

Your should find

$$\Phi(\rho, \phi) = \frac{\tau}{4\pi\epsilon_0} \ln \frac{R^2\rho^2 + b^4 - 2\rho R b^2 \cos\phi}{R^2\rho^2 + R^4 - 2\rho R^3 \cos\phi}. \tag{1}$$

This can be written as (believe it or not!)

$$\Phi(\rho, \phi) = \frac{\tau}{2\pi\epsilon_0} \sum_{n=1}^{\infty} \frac{R^{2n} - b^{2n}}{n(\rho R)^n} \cos(n\phi). \tag{2}$$

By explicit Taylor expansion of Eq. (1) in the small unitless parameter $\zeta \equiv \frac{R}{\rho} \ll 1$, using the auxiliary variable $\Lambda \equiv \frac{b}{R} < 1$ as a shorthand to simplify your algebra, derive the first 3 terms ($n = 1, 2, 3$) of Eq. (2). You will need trogonometric identities to convert $\cos^n \phi$ into a sum of $\cos(m\phi)$ terms (with $m \leq n$).