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Neutrino heat conduction and inhomogeneities in the early Universe

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Constraints on parameters of inhomogeneous nucleosynthesis, namely, the overdensity and size of baryon lumps, are found by calculating the blackbody neutrino heat conduction into the lumps, which tends to inflate them away. The scale size for efficient heat conduction is determined by the mean free path λ of the neutrino, and so we compute λ in our case of a high-temperature plasma with low chemical potential, and find a general result that many-body effects are unimportant, simplifying the calculation. We find that in the region of interest for nucleosynthesis, neutrino inflation is important for overdensities $> 10^4$.

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I. INTRODUCTION

Most calculations concerning the early Universe, such as big-bang nucleosynthesis, assume that the Universe was homogeneous. But with the likelihood that several phase transitions occurred after the big bang as the Universe cooled down, perhaps it is more accurate and more natural to assume that the Universe was significantly inhomogeneous during certain epochs [1–3]. The electroweak phase transition, for example, may have produced baryons and created overdense baryon lumps that survived until the nucleosynthesis epoch [4]. The effects of such overdense baryon lumps on primordial nucleosynthesis have been extensively investigated [1,5–7], and they can significantly affect the primordial abundances of the elements. In fact, new observations of beryllium to boron abundances might be explained by assuming inhomogeneous big-bang nucleosynthesis (IBBN) [8].

But as much as these inhomogeneities may be generated, they will also tend to smooth themselves out due to diffusion. For example, it is well known that any (small-scale) inhomogeneities present during the nucleosynthesis epoch will be completely smoothed out at cooler temperatures by photon diffusion [9,10]. There is an analogous effect at higher temperatures when the neutrino is still coupled to the plasma yet still has a long enough mean free path to conduct heat and smooth out inhomogeneities.

In this paper, we will calculate the evolution of an overdense baryon lump subject to this mechanism, called "neutrino inflation." Neutrino inflation can be more precisely described in the following way. Any overdense baryon lumps in the early Universe will quickly come to pressure equilibrium with the surroundings, but the extra baryon density will cause the temperature of the lump to be slightly lower. The neutrinos, because of their relatively long mean free path, will be the most efficient conductors of heat into the lump, inflating the lump and reducing its overdensity. Using a simple model for heat conduction, we will find an equation for the overdensity as a function of time (or temperature of the Universe), to see how the lumps evolve from 100 to 1 MeV. The results will show that certain sizes and overdensities of lumps cannot exist at 1 MeV, and this has the added benefit of reducing the allowed parameter space used in IBBN calculations.

Knowing the neutrino mean free path λ is important for setting the scale for efficient heat conduction, so in Sec. IV we calculate λ for a relativistic gas at high temperature and examine the issue of many-body effects. Note here that the effects of neutrino inflation have previously been qualitatively estimated [1,11], but the calculations in this paper give a much more accurate picture of the neutrino heat transport.

II. NEUTRINO INFLATION

We will start out by assuming a spherical overdense baryon lump that may have originated from a variety of mechanisms such as a first-order quark to hadronic-matter phase transition. The lump has a baryon density n_b much greater than the background density ($n_b \gg n_{b_0}$), mass M, and radius R. We will be concerned with temperatures between 100 and 1 MeV, that is, shortly after the quark-hadron phase transition, but (roughly) before the neutrino decouples from the plasma. In the following calculations we will neglect the presence of pions and briefly discuss their effects in the conclusion.

At 100 MeV the Universe is radiation dominated and is filled with $\mu^\pm$, $e^\pm$, ν, $\bar{\nu}$, γ, all in equilibrium. The total radiation energy density is $\rho_{\rm rad}=N_{\rm eff}aT^4$ and pressure $p_{\rm rad}=N_{\rm eff}aT^4/3$. Assume the baryons make up an ideal gas with $p_b=n_bT$.

If there is any difference in pressure between the lump and the background, then the lump will have a characteristic pressure relaxation time $ct_{\rm relax}\approx R$. So as long as $R \ll c$(Hubble time), the lump will quickly reach pressure equilibrium with the background, and we can write

$$n_bT+\tfrac{1}{3}N_{\rm eff}aT^4=\tfrac{1}{3}N_{\rm eff}aT_0^4+n_{b_0}T_0\ . \qquad (1)$$

So we can see that the temperature of the lump must be slightly less than the background temperature $T=T_0-\delta T$. To first order in δT, Eq. (1) becomes

$$\frac{\delta T}{T} \approx \frac{\eta_0}{N_{\text{eff}}} \frac{n_b}{n_{b_0}} \equiv \frac{\eta_0}{N_{\text{eff}}} \delta_n , \tag{2}$$

where δ_n is defined to be the overdensity and η_0 is the present baryon to photon ratio. Note that η is not constant because of muon and electron annihilation at 10 and 0.05 MeV. As a result, $\eta(100 < T < 10 \text{ MeV}) \approx 3.6\eta_0$, which also takes into account the fact that the neutrinos decouple at 2 MeV. We will use the middle of the road value $\eta_0 = 3 \times 10^{-9}$, which gives a fractional baryon density $\Omega_b \approx 0.13/h^2$.

Our goal is to find how the lump's overdensity evolves with time, so let us first calculate the rate of heat conduction into the lump. The way in which heat is conducted into the lump depends upon how large it is compared to the mean free path λ of the neutrino. In this section we will assume $R \approx \lambda$. We will look at larger and smaller lumps in the next section.

If $R \approx \lambda$, then the blackbody neutrino radiation in the background plasma is approximated to be perfectly absorbed throughout the lump. The energy flux into the lump is [12]

$$\Phi = \frac{1}{4} \sum_i [\rho_i(T_0) - \rho_i(T)] \approx \sum_i \rho_i(T_0) \frac{\delta T}{T} , \tag{3}$$

where i is the type of neutrino (there are $2 \times 3 = 6$ of them). Note that any sterile neutrinos will only slightly change N_{eff}, and any effects due to massive neutrinos ($m_\nu \ll T$ or $m_\nu \gg T$) will be negligible.

Therefore, using (2) and (3) we find the rate of heat conduction into the lump to be (for $R \approx \lambda$)

$$\frac{dE}{dt} = 4\pi R^2 \Phi = 4\pi R^2 6 \left[\frac{7}{16} \right] a T^4 \frac{\eta_0 \delta_n}{N_{\text{eff}}} . \tag{4}$$

Any added heat ΔE will add to the volume of the lump an amount $\Delta V \approx \Delta E / \rho_{\text{rad}}(t)$, and since the number of baryons in the lump is constant we can write

$$\delta_n(t + \Delta t) = \frac{\delta_n(t) V(t)}{\Delta E / \rho_{\text{rad}}(t) + V(t)}$$

$$\Longrightarrow \frac{d\delta_n}{dt} = - \frac{\delta_n}{V(t) \rho_{\text{rad}}(t)} \frac{dE}{dt} . \tag{5}$$

It is more convenient to speak in terms of δ_n as a function of temperature rather than time. By using the Friedmann equation we can get a relationship between time and temperature:

$$\frac{1}{T^2} = \left[\frac{32\pi}{3} G N_{\text{eff}} a \right]^{1/2} t . \tag{6}$$

Combining Eqs. (4)–(6), noting that $\frac{4}{3}\pi R^3 m_p n_{b_0} \delta_n = M$ and $n_{b_0} = (3.6/2.7)\eta_0 a T^3$, and integrating, we get a relation for overdensity as a function of temperature of the evolving Universe for a lump of mass M:

$$\frac{1}{\delta_n^{4/3}(T)} = \frac{(8 \times 10^3 \text{ MeV}) \eta_0^{4/3}}{N_{\text{eff}}^{5/2} (M/M_\odot)^{1/3}} \left[\frac{1}{T} - \frac{1}{T_0} \right] + \frac{1}{\delta_n^{4/3}(T_0)} . \tag{7}$$

Note that this equation holds for any lump whose radius $R \approx \lambda$ between the temperatures T_0 and T.

III. LARGE AND SMALL LUMPS

In the above scenario, we have assumed that the neutrinos efficiently conducted heat into the entire lump, and for this to be true, the size of the lump must be on the same order as the neutrino's mean free path λ. What if R is larger or smaller than the mean free path?

In the case $R \ll \lambda$, the neutrinos are "free streaming" in the sense that they have approximately the same temperature inside and outside the lump. On average, every neutrino crossing the lump deposits an energy

$$E_{\text{dep}} \approx \frac{R}{\lambda} \frac{\bar{E}_\nu(T_0) - \bar{E}_\nu(T)}{2} \approx \frac{R}{2\lambda} 3.15 \delta T . \tag{8}$$

The number of particles crossing the lump in time t_{inf} is $N = \pi R^2 t_{\text{inf}} \sum_i n_{\nu i}$, and so by using (2) and (8) we find the total energy deposited into the lump per unit time to be (for $R \ll \lambda$)

$$\frac{dE}{dt} \approx \frac{\pi R^3}{2\lambda} 6 \left[\frac{7}{16} \right] a T^4 \frac{\eta_0 \delta_n}{N'_{\text{eff}}} , \tag{9}$$

where N'_{eff} is the number of particle degrees of freedom in the plasma minus the neutrinos, since they are approximately free streaming through the lump. For the value of λ, we will use the relation $\lambda T^5 \approx \text{const}$, which is calculated in the next section for $T = 100$ MeV. Any deviations from a constant value (explained in the next section) will have a negligible effect on our results. After integration, Eqs. (5), (6), and (8) yield (for $R \ll \lambda$)

$$\frac{1}{\delta_n(T)} \approx \frac{3.3\eta_0}{N_{\text{eff}}^{1/2} N_{\text{eff}}'^{2} \lambda_{100}} (T^3 - T_0^3) + \frac{1}{\delta_n(T_0)} , \tag{10}$$

where λ_{100} is the mean free path of the neutrino at 100 MeV in units of cm, and T is in units of MeV.

If $R \gg \lambda$ then neutrino conduction is less important. But as the temperature decreases, the mean free path increases as $\sim 1/T^5$, and so eventually any lump smaller than the horizon size will reach the regime $R \approx \lambda$ and the neutrinos can penetrate into the entire lump. Before that time the lump's density will decrease only around the edges, and, indeed, one could even calculate the evolution of an initial "top hat" density distribution as λ gets bigger. For simplicity though, we assert that if $R > 10\lambda$ then any neutrino diffusion is negligible, and no neutrino conduction occurs. Therefore, as an approximation to find the density evolution of any lump of any size, we will use the following method. For $R > 10\lambda$, $\delta = \text{const}$; for $10\lambda < R < \lambda$, we will use Eq. (7); for $\lambda > R$ we will use Eq. (10). Note that the radius of the lump obeys the equation $R = R_0 (T_0/T)(\delta_0/\delta)^{1/3}$. That is, the lump not only expands with the Universe, but also its comoving radius increases because the neutrinos are inflating it.

The results of these calculations are shown in Fig. 1, which shows the evolution of a lump beginning at 100 MeV and given an initial overdensity and radius.

Figure 2 is a combination of the results in Fig. 1, show-

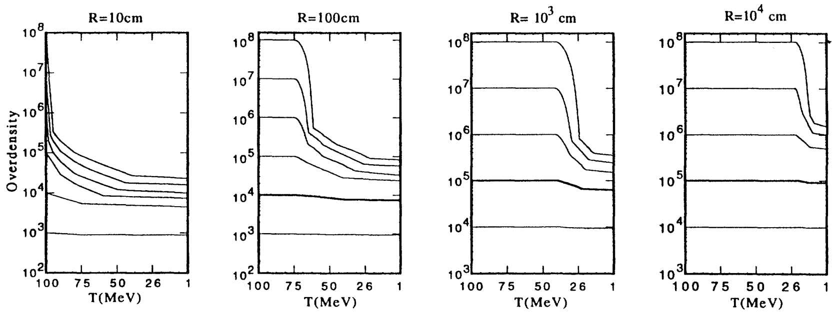


FIG. 1. Density evolution of overdense baryon lumps from 100 to 1 MeV. The radius given is the lump's initial physical radius at 100 MeV, and each line starts at the lump's initial overdensity. For larger initial radii, the lump's overdensity does not change until its size is comparable to the mean free path of the neutrino.

ing what maximum possible overdensities can finally exist at 1 MeV. The knee in the graph is sensitive to the temperature at which we started the evolution, namely, 100 MeV, because this also determines the range of sizes of λ, and therefore which lump sizes will experience efficient heat conduction (it is most efficient for $R \approx \lambda$). What if the lumps were formed earlier? We have made a crude extension of the model to higher initial temperatures, assuming that for $T > 100$ MeV the quarks in the plasma are free. The extension of the model (dotted line in Fig. 2) shows that since at higher initial temperatures λ is smaller, the smaller lumps are constrained even further. It is interesting to note that *no* overdense lumps with $R < 3\times10^{-2}$ cm can exist at 1 MeV. An equivalent statement is that at any temperatures greater than about 1 GeV, any lumps that are equal in size or smaller than the mean free path of the neutrino will be completely washed out almost immediately. It should also be noted that the extra constraints that come from the extension of this model only affect lumps that are smaller than those that are of interest in the current IBBN calculations [5,6].

IV. MEAN FREE PATH: IS THERE SCREENING?

As shown in the previous two sections, the mean free path λ sets the scale for determining how the neutrinos

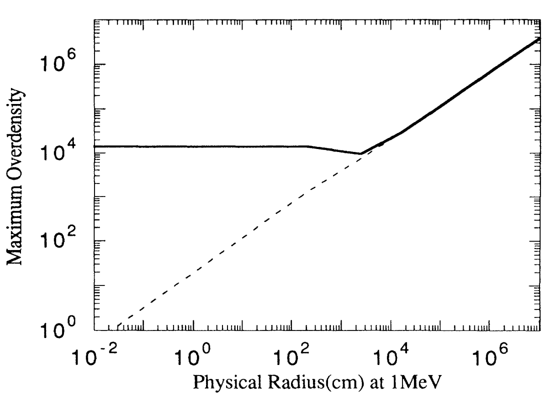


FIG. 2. Upper limits on overdensity at $T=1$ MeV. Because large overdensities are dissipated quickly, only overdensities below this line can exist at 1 MeV. The solid-line graph shows the excluded parameter space for lumps created at 100 MeV. The dotted line shows the extra excluded parameter space if the lumps were created at a much higher initial temperature ($T_0 > 2$ GeV).

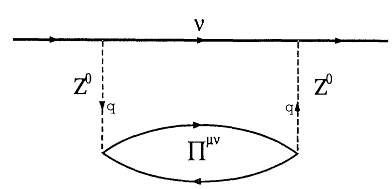
will conduct heat into the lump. What is the mean free path of a neutrino in a high-temperature plasma? We will answer this by first determining whether many-body effects are important.

When a neutrino weakly scatters, via the neutral current, off of an electron in an electron gas, the electron will recoil. But the way the electron recoils (and so the neutrino) will depend upon its interactions with the other electrons in the gas. Therefore, the cross section of a neutrino in an electron gas must include the many-body effects between the electrons. All of this information is included in the polarization propagator [14,15]. Here, we will consider only electromagnetic interactions between electrons because they are the strongest.

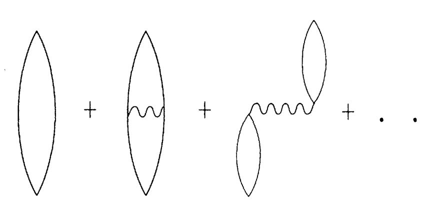
Following the notation of Horowitz and Wehrberger [16], the differential cross section for neutral-current interactions of a neutrino of initial energy E and final energy E' in an electron gas can be shown to be

$$\frac{1}{V}\frac{d^3\sigma}{d^2\Omega\, dE'} = -\frac{G_F^2}{32\pi^2}\frac{E'}{E}\,\mathrm{Im}(L_{\mu\nu}\Pi^{\mu\nu}) , \qquad (11)$$

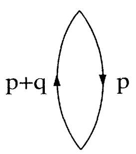
with $L_{\mu\nu}$ the neutrino part of the matrix element squared and $\Pi^{\mu\nu}$ the electron polarization propagator including vector and axial-vector vertices. Horowitz and Wehrberger calculated $\Pi^{\mu\nu}$ in the limit $k_f \gg |\mathbf{q}|$ or $T \gg |\mathbf{q}|$, where $|\mathbf{q}|$ is the momentum transfer (high-density limit), and showed that screening effects are important. In our case, however, the chemical potential is zero and the electrons and neutrinos are thermal, giving $|\mathbf{q}| \approx T$. Fortunately, the polarization propagator is very simple in this case. Schematically, the imaginary part of $L_{\mu\nu}\Pi^{\mu\nu}$ is

$\mathrm{Im}(L_{\mu\nu}\Pi^{\mu\nu}) =$  ,

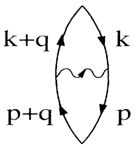
and so we want to find the important terms in the expansion of $\Pi^{\mu\nu}$:

$\Pi^{\mu\nu} =$  .

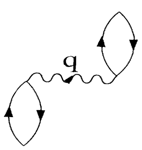
In the limit of high temperature and $|\mathbf{q}|\approx T$, we can roughly estimate the relative contribution of each diagram. The density-dependent part of each diagram goes as



$$\sim \int \frac{d^4p}{p(p+q)} \frac{1}{e^{\beta p_0}+1} \sim T^2 \,,$$



$$\sim e^2 \int \frac{d^4p}{p(p+q)} \frac{d^4k}{k(k+q)} \frac{1}{e^{\beta p_0}+1} \sim e^2T^2 \,,$$



$$\sim \frac{e^2}{q^2}\left[\int \frac{d^4p}{p(p+q)} \frac{1}{e^{\beta p_0}+1} \right]^2 \sim \frac{e^2}{q^2}T^4 \approx e^2T^2 \,,$$

and in our limit, the polarization propagator becomes

$$\Pi^{\mu\nu} \sim \left[1+e^2+e^2\frac{T^2}{q^2}+\cdots \right] T^2 \approx (1+e^2+\cdots)T^2 \,. \tag{12}$$

Note that normally q and μ scale differently than T, making $\Pi^{\mu\nu}$ a diverging or slowly converging series. But in our limit T is the only scale, and so we can expand $\Pi^{\mu\nu}$ in powers of e^2. Therefore, to an accuracy $\sim e^2$ we can ignore all diagrams other than the single particle-hole bubble. Put another way, to an excellent approximation we can assume the gas is noninteracting and there are no screening effects. This is a general result: If we quasielastically scatter a particle off of a relativistic gas of temperature $T \gg \mu$ and $|\mathbf{q}|\approx T$, then to order g^2 we can assume the gas is noninteracting and no screening occurs. Here g is the coupling between the particles of the gas [17]. This result also has the simple interpretation that since the kinetic energy of the neutrino is about $1/\alpha$ times greater than the average potential energy between the electrons, we can ignore the interactions between the electrons.

This makes calculation of the mean free path easy because now we can assume that the neutrino scatters incoherently off of one particle at a time. This is a well-known limit.

For simplicity, let us first assume that we have a gas of electrons and neutrinos at temperature T. Then the rate of $\nu e \rightarrow \nu e$ collisions suffered by a typical neutrino with momentum k is defined to be [18]

$$R = \frac{1}{2E_k}\int \frac{2d^3p}{(2\pi)^3 2E_p} \frac{d^3k'}{(2\pi)^3 2E_{k'}} \frac{d^3p'}{(2\pi)^3 2E_{p'}} \times [(2\pi)^4\delta^4(k+p-k'-p') \times \overline{|\mathcal{M}|^2} f_p(1-f_{k'})(1-f_{p'})] \,. \tag{13}$$

TABLE I. Mean free paths of neutrinos in the early Universe [excluding pions and other neutrinos (see text)]. There is almost an exact T^5 dependence, but there are no $\mu^\pm$ or $\tau^\pm$ at 1 MeV, so the mean free paths will be slightly longer.

	$T=100$ MeV	$T=1$ MeV
$\lambda_{\nu_e,\bar{\nu}_e}$	2.3 cm	5.5×10^{10} cm
$\lambda_{\nu_\mu,\bar{\nu}_\mu}$	2.3 cm	22×10^{10} cm
$\lambda_{\nu_\tau,\bar{\nu}_\tau}$	11 cm	22×10^{10} cm

We will assume that all chemical potentials are zero, and at large temperatures we will take as a good approximation that $1-f_{k'}\approx 1-f_{p'}\approx 1$, where f_p is the fermion occupation number at momentum p. In the relativistic limit, Tubbs and Schramm [15] have shown that the rate of νe collisions is

$$R = \frac{G_F^2}{2\pi}[(g_V'g_A')^2+\tfrac{1}{3}(g_V'-g_A')^2]\tfrac{4}{3}\bar{E}_\nu\rho_e \,, \tag{14}$$

where $g'=g+1$ is the vector (or axial-vector) coupling which includes both charged and neutral currents, $\bar{E}_\nu$ is the average energy of the neutrino, and ρ_e is the energy density of the electrons. Similar expressions arise for gases of $\mu^\pm$, e^+, ν, and $\bar{\nu}$.

We must be careful about finding the total cross section of the neutrino in the plasma. Recall that we are interested in lumps small enough that neutrinos almost free stream through them. If we want the neutrinos to deposit energy into the lump, they must do so via νe or $\nu\mu$ collisions. Any $\nu\nu\rightarrow\nu\nu$ will not affect such small lumps ($R<\lambda$), so in computing λ we will exclude $\nu\nu$ collisions. The only exception is $\nu\bar{\nu}\rightarrow e^+e^-, \nu\bar{\nu}\rightarrow\mu^+\mu^-$, and we will include these.

Therefore, the mean free path λ of a neutrino in a plasma of all above possible particles (ignoring hadrons) is

$$\lambda = (R_{\rm tot})^{-1} = \left[\frac{G_F^2}{2\pi}\frac{4}{3}\bar{E}_\nu \sum_i A_i\rho_i \right]^{-1} \,, \tag{15}$$

where A_i is the coupling coefficient for particle i. Assuming all particles to be relativistic, the average rate of collisions of a typical ν_e or $\bar{\nu}_e$ is found to be

$$R \simeq 3.1 G_F^2 T^5 \quad \text{at } T=100 \text{ MeV} \,,$$

$$R \simeq 1.3 G_F^2 T^5 \quad \text{at } T=1 \text{ MeV} \,.$$

Similar expressions can be calculated for the other two neutrino flavors, and the mean free paths are in Table I. The τ neutrino has a larger mean free path because it does not participate in any charged-current scattering in the absence of τ's. Since for small lumps the energy deposited goes as $1/\lambda$, we used an averaged value $1/\lambda = \frac{1}{3}\sum_i 1/\lambda_i$ for the mean free path.

V. CONCLUSIONS

This calculation is only a simple model for neutrino inflation. For example, we have assumed that all of the muons instantaneously annihilate at 10 MeV, when actually they slowly annihilate from 100 to 1 MeV. Also we have ignored the presence of pions which will increase $N_{\rm eff}$ and decrease the mean free path of the neutrino. But these added effects from the muons and pions will partially cancel, and rough calculations show the error to be well within the built-in uncertainty of this simple model for heat conduction (namely, that we did not solve exactly by using the Boltzmann equation), which is probably as much as a factor of 2. The main point is that we have used standard physics to show that neutrino heat conduction effects cannot be ignored for inhomogeneities in the Universe above 1 MeV, and in particular, for lump sizes of interest to IBBN, neutrino inflation is important for overdensities $>10^4$. Therefore one is not free to choose any values in parameter space for lumps at 1 MeV.

As a second point of this paper, we have found that for a relativistic thermal gas, to a very good approximation the particles do not participate in any screening effects when they scatter, and this simple result is useful for calculating the mean free path, and thus heat conduction, in such a gas.

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