Direct and Indirect $CP$ violation

- $CP$ is not conserved in neutral kaon decay.
  - It makes more sense to use mass (or lifetime) eigenstates rather than $|K_1>$ and $|K_2>$:
    - “$K$-short”: short lifetime state with $\tau_s \approx 9 \times 10^{-11}$ sec.
      \[
      |K_S\rangle = \frac{1}{\sqrt{1 + |\epsilon|^2}} \left(|K_1\rangle + \epsilon |K_2\rangle\right)
      \]
    - “$K$-long”: long lifetime state with $\tau_L \approx 5 \times 10^{-8}$ sec.
      \[
      |K_L\rangle = \frac{1}{\sqrt{1 + |\epsilon|^2}} \left(|K_2\rangle + \epsilon |K_1\rangle\right)
      \]
    - $\epsilon$ is a (small) complex number that allows for $CP$ violation through mixing.

- There are two types of $CP$ violation in $K_L$ decay:
  - Indirect (“mixing”): $K_L \to \pi\pi$ because of its $K_1$ component
  - Direct: $K_L \to \pi\pi$ because the amplitude for $K_2$ allows $K_2 \to \pi\pi$
  - Experimental observation: indirect $>>$ direct!

- The strong interaction eigenstates with definite strangeness are:
  - $K^0 = |\bar{s}d\rangle$ and $\bar{K}^0 = |sd\rangle$
  - Consider the strangeness operator:
    \[
    S|K^0\rangle = |K^0\rangle \quad \text{and} \quad S|\bar{K}^0\rangle = -|\bar{K}^0\rangle
    \]
    - They are particle and anti-particle and by the $CPT$ theorem have the same mass.
      - Experimentally we find:
        \[
        (m_{K^0} - m_{\bar{K}^0})/m < 9 \times 10^{-19}
        \]
The $K_L$ and $K_S$ states are not $CP$ (or $S$) eigenstates:

$$CP|K_S\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}} (CP|K_1\rangle + \varepsilon CP|K_2\rangle) = \frac{1}{\sqrt{1+|\varepsilon|^2}} (|K_1\rangle - \varepsilon |K_2\rangle) \neq |K_S\rangle$$

$$CP|K_L\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}} (CP|K_2\rangle + \varepsilon CP|K_1\rangle) = \frac{1}{\sqrt{1+|\varepsilon|^2}} (-|K_2\rangle + \varepsilon |K_1\rangle) \neq |K_L\rangle$$

- In fact these states are not orthogonal:

$$\langle K_S | K_L \rangle = \frac{2 \text{Re} \varepsilon}{1+|\varepsilon|^2} \neq 0$$

- If $CP$ violation is due to mixing (“indirect” only):
  - The amplitude for $K_L \rightarrow \pi\pi$:
    $$\langle K_1 | K_L \rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}} (\langle K_1 | K_2 \rangle + \varepsilon \langle K_1 | K_1 \rangle) = \frac{\varepsilon}{\sqrt{1+|\varepsilon|^2}}$$
  - The amplitude for $K_L \rightarrow \pi\pi\pi$:
    $$\langle K_2 | K_L \rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}} (\langle K_2 | K_2 \rangle + \varepsilon \langle K_2 | K_1 \rangle) = \frac{1}{\sqrt{1+|\varepsilon|^2}}$$

- Experimental measurement: $|\varepsilon| = 2.3 \times 10^{-3}$.
- The standard model predicts a small amount of direct $CP$ violation too!
Direct $CP$ violation

- Standard model predicts that quantities $\eta_{+-}$ and $\eta_{00}$ should differ very slightly due to direct $CP$ violation:

$$
\eta_{+-} = \frac{\text{Amp}(K_L \rightarrow \pi^+\pi^-)}{\text{Amp}(K_S \rightarrow \pi^+\pi^-)} \quad \eta_{00} = \frac{\text{Amp}(K_L \rightarrow \pi^0\pi^0)}{\text{Amp}(K_S \rightarrow \pi^0\pi^0)}
$$

- This is $CP$ violation in the amplitude.

- $CP$ violation is now described by two complex parameters, $\epsilon$ and $\epsilon'$:
  - $\epsilon'$ is related to direct $CP$ violation.
  - The standard model estimates:
    \[
    \text{Re}(\frac{\epsilon'}{\epsilon}) \sim (4-30) \times 10^{-4}
    \]
  - Experimentally what is measured is the ratio of branching ratios:
    \[
    \frac{\text{BR}(K_L \rightarrow \pi^+\pi^-)}{\text{BR}(K_S \rightarrow \pi^+\pi^-)} \frac{\text{BR}(K_L \rightarrow \pi^0\pi^0)}{\text{BR}(K_S \rightarrow \pi^0\pi^0)} = \left| \frac{\eta_{+-}}{\eta_{00}} \right|^2 = \left| \frac{\epsilon + \epsilon'}{\epsilon - 2\epsilon'} \right|^2 \approx 1 + 6 \text{Re}\left( \frac{\epsilon'}{\epsilon} \right)
    \]

- There were multiple attempts to measure the ratio since 1970's with some controversial results.
  - A non-zero value has recently been measured by 2 different experiments:
    \[
    \text{Re}(\frac{\epsilon'}{\epsilon}) = (17.2 \pm 1.8) \times 10^{-4}
    \]
  - Currently the measurement is more precise than the theoretical calculation!
  - Calculating $\text{Re}(\frac{\epsilon'}{\epsilon})$ is presently one of the most challenging HEP theory projects.
Neutral Kaons and Strangeness Oscillations

- We now consider how the neutral kaon state evolves with time:
  - allow us to measure $K_L$, $K_S$ mass difference and phases of the $CP$ violation parameters, $\eta_{+-}$ and $\eta_{00}$.
- How a two-particle quantum system evolves in time is covered in many texts including:
  - *Particle Physics, Martin and Shaw, section 10.3*
  - *Introduction to High Energy Physics, Perkins*
  - *Introduction to Nuclear and Particle Physics, Das and Ferbel*
  - *Lectures on Quantum Mechanics, Baym, Ch 2.*
  - *The Feynman Lectures, Vol III, section 11-5.*
    - Written in about 1963, before there was good experimental data on this topic!
    - $CP$ violation was discovered in 1964.
- The following derivation is for the neutral kaon system.
  - It is also applicable to $B$-meson and neutrino oscillations.
Kaon Oscillations

- First, consider the case where CP is conserved.
  - CP eigenstates \(|K_1>| and \(|K_2>| are solutions to the time dependent Schrödinger equation:
    \[ i\hbar \frac{\partial |\psi(t)\rangle}{\partial t} = H_{\text{eff}} |\psi(t)\rangle \]
  - \(H_{\text{eff}}\) is a phenomenological Hamiltonian that describes the system.
  - Since our particles can decay, \(H_{\text{eff}}\) is not a Hermitian operator!
  - Since there are two states in this problem it is customary:
    - Describe \(H_{\text{eff}}\) by a 2x2 matrix (e.g. see Das and Ferbel Chapter XII).
    - Describe \(|K_1>| and \(|K_2>| by column vectors.
  - \(H_{\text{eff}}\) matrix is written in terms of the masses \((m_1, m_2)\) and lifetimes \((\tau_1, \tau_2)\) of the two states:
    \[
    H_{\text{eff}} = \begin{pmatrix}
    \frac{1}{2} (m_1 + m_2) - \frac{i}{4} \hbar \left( \frac{1}{\tau_1} + \frac{1}{\tau_2} \right) & \frac{1}{2} (m_2 - m_1) - \frac{i}{4} \hbar \left( \frac{1}{\tau_2} - \frac{1}{\tau_1} \right) \\
    \frac{1}{2} (m_2 - m_1) - \frac{i}{4} \hbar \left( \frac{1}{\tau_2} - \frac{1}{\tau_1} \right) & \frac{1}{2} (m_1 + m_2) - \frac{i}{4} \hbar \left( \frac{1}{\tau_1} + \frac{1}{\tau_2} \right)
    \end{pmatrix}
    \]
  - The eigenvalues and eigenvectors of \(H_{\text{eff}}\) are:
    \[
    \begin{align*}
    H_{\text{eff}} |K_1(t)\rangle &= (m_1 - \frac{i\hbar}{2\tau_1}) |K_1(t)\rangle \quad \text{with } |K_1(t)\rangle = \begin{pmatrix} +f_1(t) \\ -f_1(t) \end{pmatrix} \\
    H_{\text{eff}} |K_2(t)\rangle &= (m_2 - \frac{i\hbar}{2\tau_2}) |K_2(t)\rangle \quad \text{with } |K_2(t)\rangle = \begin{pmatrix} +f_2(t) \\ -f_2(t) \end{pmatrix}
    \end{align*}
    \]
  - The states are orthogonal to each other: \(<K_1(t)|K_2(t)> = <K_2(t)|K_1(t)> = 0.\]

[\text{f}(t)\text{ contains the time dependence.}]

- The solutions to the Hamiltonian:
  \[
  |K_1(t)\rangle = e^{-\frac{i}{\hbar} \left( \frac{m_1}{2\tau_1} - \frac{i\hbar}{2\tau_1} \right) t} |K_1\rangle \\
  |K_2(t)\rangle = e^{-\frac{i}{\hbar} \left( \frac{m_2}{2\tau_2} - \frac{i\hbar}{2\tau_2} \right) t} |K_2\rangle
  \]
Neutral Kaons and Strangeness Oscillations

- Consider an experiment which produces a beam of pure $K^0$'s (at $t = 0$) using the strong interaction: $\pi p \rightarrow \Lambda K^0$
- As in the previous lecture, we can express a $K^0$ as of a mixture of $|K_1>$ and $|K_2>$ (and visa versa):
  $$|K_1\rangle = \frac{1}{\sqrt{2}} \left( |K^0\rangle + |\bar{K}^0\rangle \right) \quad |K_2\rangle = \frac{1}{\sqrt{2}} \left( |K^0\rangle - |\bar{K}^0\rangle \right)$$
  $$|K^0\rangle = \frac{1}{\sqrt{2}} \left( |K_1\rangle + |K_2\rangle \right) \quad |\bar{K}^0\rangle = \frac{1}{\sqrt{2}} \left( |K_1\rangle - |K_2\rangle \right)$$
- Using the time dependent solutions for $K_1$ and $K_2$ we can find the time dependent solution for $K^0$:
  $$|K^0(t)\rangle = \frac{1}{\sqrt{2}} \left( e^{\frac{-i}{\hbar}(m_1-\frac{ih}{2\tau_1})t} |K_1\rangle + e^{\frac{-i}{\hbar}(m_2-\frac{ih}{2\tau_2})t} |K_2\rangle \right)$$
- The amplitude for finding a $K^0$ in the beam at a later time ($t$) is given by:
  $$\langle K^0 | K^0(t) \rangle = \frac{1}{2} \left( \langle K_1 | + \langle K_2 | \right) e^{\frac{-i}{\hbar}(m_1-\frac{ih}{2\tau_1})t} |K_1\rangle + e^{\frac{-i}{\hbar}(m_2-\frac{ih}{2\tau_2})t} |K_2\rangle$$
  $$= \frac{1}{2} \left( e^{\frac{-i}{\hbar}(m_1-\frac{ih}{2\tau_1})t} + e^{\frac{-i}{\hbar}(m_2-\frac{ih}{2\tau_2})t} \right)$$
Neutral Kaons and Strangeness Oscillations

The probability to find a $K^0$ in the beam at a later time is given by:

\[
\left| \left\langle K^0 \left| K^0(t) \right. \right\rangle \right|^2 = \frac{1}{4} \left( e^{\frac{i}{\hbar} (m_1 + i\hbar \frac{t}{2\tau_1})} + e^{\frac{i}{\hbar} (m_2 + i\hbar \frac{t}{2\tau_2})} + e^{\frac{i}{\hbar} (m_1 - i\hbar \frac{t}{2\tau_1})} + e^{\frac{i}{\hbar} (m_2 - i\hbar \frac{t}{2\tau_2})} \right)
\]

\[
= \frac{1}{4} \left( e^{-\frac{t}{\tau_1}} + e^{-\frac{t}{\tau_2}} + e^{\frac{i}{\hbar} (m_1 - m_2 + i\hbar \frac{t}{2\tau_1})} + e^{\frac{i}{\hbar} (m_1 - m_2 - i\hbar \frac{t}{2\tau_2})} \right)
\]

\[
= \frac{1}{4} \left( e^{-\frac{t}{\tau_1}} + e^{-\frac{t}{\tau_2}} + e^{\frac{i}{\hbar} (m_2 - m_1 + i\hbar \frac{t}{2\tau_1})} + e^{\frac{i}{\hbar} (m_2 - m_1 - i\hbar \frac{t}{2\tau_2})} \right)
\]

\[
= \frac{1}{4} \left( e^{-\frac{t}{\tau_1}} + e^{-\frac{t}{\tau_2}} + e^{\frac{i}{\hbar} (m_2 - m_1 + \frac{t}{2\tau_1})} + e^{\frac{i}{\hbar} (m_2 - m_1 - \frac{t}{2\tau_2})} \right)
\]

\[
= \frac{1}{4} \left( e^{-\frac{t}{\tau_1}} + e^{-\frac{t}{\tau_2}} + 2 e^{-\frac{t}{2\tau_1}} \cos \left( \frac{t}{\hbar} (m_2 - m_1) \right) \right)
\]

To make the units come out right in the cos term substitute $(m_2-m_1)c^2$ for $(m_2-m_1)$.

- The third term in the above equation is an interference term.
- It causes an oscillation that depends on the mass difference between $|K_1>$ and $|K_2>$.
- Observation of neutrino oscillation implies that neutrinos must have mass.

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L7: CP Violation
Flavor Oscillations

- Using the same procedure, we can calculate the probability that a beam initially consisting of $K^0$'s contains $\bar{K}^0$'s at a later time:

$$\left|\langle \bar{K}^0 | K^0 (t) \rangle \right|^2 = \frac{1}{4} \left( e^{-\frac{t}{\tau_1}} - e^{-\frac{t}{\tau_2}} - 2e^{-\frac{t}{2(\frac{1}{\tau_1} + \frac{1}{\tau_2})}} \cos \left(\frac{t}{\hbar} (m_2 - m_1)\right) \right)$$

- The sign of the interference term is now “-”.
- Since the strangeness of a $K^0$ differs from a $\bar{K}^0$'s, the strangeness content of the beam is changing as a function of time.
  - This phenomena is called *strangeness* oscillations.
  - More generally we call this phenomena *flavor* oscillations:
    - It also occurs with $B$-mesons ($b$ quark oscillations) and neutrinos (e.g. $\nu_e \Leftrightarrow \nu_\mu$).

- We can measure the strangeness content of a beam as a function of time (distance):
  - put some material in the beam
  - counting the number of strong interactions with $S = +1$ in the final state vs. number with $S = -1$:
    - $K^0 \ p \rightarrow K^+ n \quad S = +1$
    - $\bar{K}^0 \ p \rightarrow \Lambda p \quad S = -1$

---

Strangeness oscillations for an initially pure $K^0$ beam. A value of $(m_2 - m_1)\tau_1 = 0.5$ is used.
**CP Violation**

- **CP** violation requires modifying $H_{\text{eff}}$ to include additional complex parameters.
  - The details of the derivation are given in many texts.
    - e.g. *Weak Interactions of Leptons and Quarks* by Commins and Bucksbaum.

- **CP** violation can be measured using the following procedure:
  - produce a beam that is $K^0$ initially
  - measure the yield of $\pi^+\pi^-$ decays as a function of proper time, time measured in the rest frame of $K^0$.
    - This is a measure of the sum of the square of the amplitude $|K_L \to \pi^+\pi^-|$ and $|K_S \to \pi^+\pi^-|$.
    - There will be an interference term in the number of $\pi^+\pi^-$ decays per unit time ($\equiv I(t)$).
  - The yield of $\pi^+\pi^-$ decays is given by:
    \[
    I_{\pi^+\pi^-}(t) = I_{\pi^+\pi^-}(0) \left( e^\frac{-t}{\tau_s} + |\eta_{+-}|^2 e^\frac{-t}{\tau_L} + 2|\eta_{+-}| e^\frac{-t}{2(\tau_s + \tau_L)} \cos\left( \frac{t}{\hbar} (m_2 - m_1) + \phi_{+-} \right) \right)
    \]
  - measuring this yield provides information on:
    - the mass difference and the **CP** violation parameters $\eta_{+-}^+$ and $\phi_{+-}^+$.

Event rate for $\pi^+\pi^-$ decays as a function of proper time. The best fit requires interference between the $K_L$ and $K_S$ amplitudes:

- $m_2 - m_1 = (3.491 \pm 0.009)x10^{-6}$ eV
- $|\eta_{+-}| = (2.29 \pm 0.01)x10^{-3}$
- $\phi_{+-} = (43.7 \pm 0.6)^0$
- $\tau_S = 0.893x10^{-10}$ sec
- $\tau_L = 0.517x10^{-7}$ sec