The parity operator acting on a wavefunction:

\[ P \Psi(x, y, z) = \Psi(-x, -y, -z) \]

\[ P^2 \Psi(x, y, z) = P \Psi(-x, -y, -z) = \Psi(x, y, z) \]

☞ \( P^2 = I \)
☞ Parity operator is unitary.

If the interaction Hamiltonian \( (H) \) conserves parity

☞ \( [H,P] = 0 \)
☞ \( P_i = P_f \)

What is the eigenvalue \( (P_a) \) of the parity operator?

\[ P \Psi(x, y, z) = \Psi(-x, -y, -z) = P_a \Psi(x, y, z) \]
\[ P^2 \Psi(x, y, z) = P_a P \Psi(x, y, z) = (P_a)^2 \Psi(x, y, z) = \Psi(x, y, z) \]
\[ P_a = 1 \text{ or } -1 \]
☞ The quantum number \( P_a \) is called the intrinsic parity of a particle.
  ◆ If \( P_a = 1 \) the particle has even parity.
  ◆ If \( P_a = -1 \) the particle has odd parity.

If the overall wavefunction of a particle (or system of particles) contains spherical harmonics
☞ we must take this into account to get the total parity of the particle (or system of particles).

For a wavefunction containing spherical harmonics:

\[ P \Psi(r, \theta, \phi) = PR(r)Y_m^l (\theta, \phi) = (-1)^l R(r)Y_m^l (\theta, \phi) \]
☞ The parity of the particle: \( P_a (-1)^l \)
★ Parity is a multiplicative quantum number.

Parity Operator and Eigenvalue
Parity of Particles

- The parity of a state consisting of particles $a$ and $b$:
  \[ (-1)^L P_a P_b \]
  - $L$ is their relative orbital momentum.
  - $P_a$ and $P_b$ are the intrinsic parity of the two particles.
  - Strictly speaking parity is only defined in the system where the total momentum $p = 0$ since the parity operator ($P$) and momentum operator anticommute, $Pp = -p$.

- How do we know the parity of a particle?
  - By convention we assign positive intrinsic parity (+) to spin 1/2 fermions:
    +parity: proton, neutron, electron, muon ($\mu$)
    - Anti-fermions have opposite intrinsic parity.
  - Bosons and their anti-particles have the same intrinsic parity.
  - What about the photon?
    - Strictly speaking, we can not assign a parity to the photon since it is never at rest.
    - By convention the parity of the photon is given by the radiation field involved:
      - electric dipole transitions have + parity.
      - magnetic dipole transitions have – parity.
  - We determine the parity of other particles ($\pi$, $K$...) using the above conventions and assuming parity is conserved in the strong and electromagnetic interaction.
    - Usually we need to resort to experiment to determine the parity of a particle.
Parity of Pions

- Example: determination of the parity of the $\pi$ using $\pi d \rightarrow nn$.
  - For this reaction we know many things:
    - $s_\pi = 0$, $s_n = 1/2$, $s_d = 1$, orbital angular momentum $L_d = 0$, $J_d = 1$
    - We know (from experiment) that the $\pi$ is captured by the $d$ in an $s$-wave state.
      - The total angular momentum of the initial state is just that of the $d$ ($J = 1$).
    - The isospin of the $nn$ system is 1 since $d$ is an isosinglet and the $\pi$ has $I = |1,-1>$
      - $|1,-1>$ is symmetric under the interchange of particles. (see below)
    - The final state contains two identical fermions
      - Pauli Principle: wavefunction must be anti-symmetric under the exchange of the two neutrons.
  - Let’s use these facts to pin down the intrinsic parity of the $\pi$.
    - Assume the total spin of the $nn$ system $= 0$.
      - The spin part of the wavefunction is anti-symmetric:
        $$|0,0> = (2)^{-1/2}[|1/2,1/2>|1/2-1/2> + |1/2,-1/2>|1/2,1/2>$$
        - To get a totally anti-symmetric wavefunction $L$ must be even (0, 2, 4…)
        - Cannot conserve momentum ($J = 1$) with these conditions!
    - Assume the total spin of the $nn$ system $= 1$.
      - The spin part of the wavefunction is symmetric:
        $$|1,1> = |1,1/2,1/2>|1/2,1/2>$$
        $$|1,0> = (2)^{-1/2}[|1/2,1/2>|1/2-1/2> + |1/2,-1/2>|1/2,1/2>$$
        $$|1,-1> = |1/2,-1/2>|1/2,-1/2>$$
        - To get a totally anti-symmetric wavefunction $L$ must be odd (1, 3, 5…)
      - $L = 1$ consistent with angular momentum conservation: $nn$ has $s = 1$, $L = 1$, $J = 1 \rightarrow ^3P_1$
      - Parity of the final state: $P_n P_n (-1)^L = (+)(+)(-1)^1 = -$
      - Parity of the initial state: $P_\pi P_d (-1)^L = P_\pi (+)(-1)^0 = P_\pi$
      - Parity conservation: $P_\pi = -$
Spin Parity of Particles

- There is other experimental evidence that the parity of the $\pi$ is -:
  - The reaction $\pi d \rightarrow nn\pi^0$ is not observed.
  - The polarization of $\gamma$'s from $\pi^0 \rightarrow \gamma\gamma$.

- Spin-parity of some commonly known particles:

<table>
<thead>
<tr>
<th>State</th>
<th>Spin</th>
<th>Parity</th>
<th>Particle</th>
</tr>
</thead>
<tbody>
<tr>
<td>pseudoscalar (0⁻)</td>
<td>0</td>
<td>-</td>
<td>$\pi, K$</td>
</tr>
<tr>
<td>scalar (0⁺)</td>
<td>0</td>
<td>+</td>
<td>$a_0, \text{Higgs (not observed yet)}$</td>
</tr>
<tr>
<td>vector (1⁻)</td>
<td>1</td>
<td>-</td>
<td>$\gamma, \rho, \omega, \phi, \psi, \Upsilon$</td>
</tr>
<tr>
<td>pseudovector (axial vector) (1⁺)</td>
<td>1</td>
<td>+</td>
<td>$a_1$</td>
</tr>
</tbody>
</table>
**θ–τ Puzzle**

- How well is parity conserved?
  - Very well in strong and electromagnetic interactions ($10^{-13}$)
  - Not at all in the weak interaction!

- In the mid-1950’s it was noticed that there were 2 charged particles that had (experimentally) consistent masses, lifetimes, and spin = 0, but very different weak decay modes:
  - $\theta^+ \rightarrow \pi^+ \pi^0$
  - $\tau^+ \rightarrow \pi^+ \pi^\pm \pi^+$

  - The parity of $\theta^+ = +$ while the parity of $\tau^+ = -$.
  - Some physicists said the $\theta^+$ and $\tau^+$ were different particles, and parity was conserved.
  - Lee and Yang said they were the same particle but parity was not conserved in weak interaction!
    - Awarded Nobel Prize when parity violation was discovered.
Parity Violation in $\beta$-decay

- Classic experiment of Wu et. al. (Phys. Rev. V105, Jan. 15, 1957) looked at $\beta$ spectrum:

$$^{60}_{27}Co \rightarrow ^{60}_{28}Ni^* + e^- + \nu_e$$
$$^{60}_{28}Ni^* \rightarrow ^{60}_{28}Ni^* + \gamma(1.17) + \gamma(1.33)$$

- Parity transformation reverses all particle momenta while leaving spin angular momentum ($r \times p$) unchanged.
  - Parity invariance requires equal rate for (a) and (b).
  - Fewer electrons are emitted in the forward hemisphere.
    - An forward-backward asymmetry in the decay.
  - 3 other papers reporting parity violation published within a month of Wu et. al.!
Charge Conjugation

- Charge Conjugation \((C)\) turns particles into anti-particles and visa versa.
  
  \[
  \begin{align*}
  C(\text{proton}) & \rightarrow \text{anti-proton} & C(\text{anti-proton}) & \rightarrow \text{proton} \\
  C(\text{electron}) & \rightarrow \text{positron} & C(\text{positron}) & \rightarrow \text{electron}
  \end{align*}
  \]

- The operation of Charge Conjugation changes the sign of all intrinsic additive quantum numbers:
  - electric charge, baryon #, lepton #, strangeness, etc.
  - Variables such as momentum and spin do not change sign under \(C\).

- The eigenvalues of \(C\) are \(\pm 1\):
  \[
  \begin{align*}
  C|\psi\rangle &= |\bar{\psi}\rangle = C_\psi |\psi\rangle \\
  C^2|\psi\rangle &= C^2_\psi |\psi\rangle = |\psi\rangle \\
  \Rightarrow C^2_\psi &= 1
  \end{align*}
  \]
  - \(C_\psi\) is sometimes called the “charge parity” of the particle.
  - Like parity, \(C_\psi\) is a multiplicative quantum number.
  - If an interaction conserves \(C\)
    \[ [H,C]|\psi\rangle = 0 \]
    - Strong and electromagnetic interactions conserve \(C\).
    - Weak interaction violates \(C\) conservation.
Most particles are not eigenstates of $C$.

Consider a proton with electric charge $q$.

Let $Q$ be the charge operator:

$Q|q\rangle = q|q\rangle$

$CQ|q\rangle = qC|q\rangle = q|-q\rangle$

$QC|q\rangle = Q|-q\rangle = -q|-q\rangle$

$[C,Q]|q\rangle = [CQ-QC]|q\rangle = 2q|-q\rangle$

$C$ and $Q$ do not commute unless $q = 0$.

We get the same result for all additive quantum numbers!

Only particles that have all additive quantum numbers $= 0$ are eigenstates of $C$.

e.g. $\gamma, \rho, \omega, \phi, \psi, Y$

These particle are said to be “self conjugate”.
Charge Conjugation of Photon and $\pi^0$

- How do we assign $C_\psi$ to particles that are eigenstates of $C$?
  - Photon: Consider the interaction of the photon with the electric field.
    - As we previously saw the interaction Lagrangian of a photon is:
      $$L_{EM} = J_u A_u$$
      - $J_u$ is the electromagnetic current density and $A_u$ the vector potential.
    - By definition, $C$ changes the sign of the EM field.
      - In QM, an operator transforms as:
        $$CJC^{-1} = -J$$
    - Since $C$ is conserved by the EM interaction:
      $$CL_{EM}C^{-1} = L_{EM}$$
      $$CJ_u A_u C^{-1} = J_u A_u$$
      $$CJ_u C^{-1} CA_u C^{-1} = -J_u C A_u C^{-1} = J_u A_u$$
      $$C A_u C^{-1} = -A_u$$
    - The photon (as described by $A$) has $C = -1$.
      - A state that is a collection of $n$ photons has $C = (-1)^n$.

- $\pi^0$: Experimentally we find that the $\pi^0$ decays to 2 $\gamma$s and not 3 $\gamma$s.
  - $$\frac{BR(\pi^0 \rightarrow \gamma\gamma)}{BR(\pi^0 \rightarrow \gamma\gamma)} < 4 \times 10^{-7}$$
  - This is an electromagnetic decay so $C$ is conserved:
    $$C_\pi = (-1)^2 = +1$$
  - Particles with the same quantum numbers as the photon ($\gamma$, $\rho$, $\omega$, $\phi$, $\psi$, $Y$) have $C = -1$.
  - Particles with the same quantum numbers as the $\pi^0$ ($\eta$, $\eta'$) have $C = +1$. 
Handedness of Neutrinos

- Assuming massless neutrinos, we find experimentally:
  - All neutrinos are left handed.
  - All anti-neutrinos are right handed.
  - Left handed: spin and $z$ component of momentum are anti-parallel.
  - Right handed: spin and $z$ component of momentum are parallel.
- This left/right handedness is illustrated in $\pi^+ \rightarrow l^+ \nu_l$ decay:
  \[
  \frac{BR(\pi^+ \rightarrow e^+ \nu_e)}{BR(\pi^+ \rightarrow \mu^+ \nu_\mu)} = 1.23 \times 10^{-4}
  \]
  - If neutrinos were not left handed, the ratio would be > 1!

- Angular momentum conservation forces the charged lepton ($e$, $\mu$) to be in “wrong” handed state:
  - a left handed positron ($e^+$).
  - The probability to be in the wrong handed state $\sim m_l^2$

\[
\frac{BR(\pi^+ \rightarrow e^+ \nu_e)}{BR(\pi^+ \rightarrow \mu^+ \nu_\mu)} = \frac{m_e^2}{m_\mu^2} \frac{m_\pi^2 - m_e^2}{m_\pi^2 - m_\mu^2} \approx (1.230 \pm 0.004) \times 10^{-4}
\]

- Handedness $\sim 2 \times 10^{-5}$
- Phase space $\sim 5$
Charge Conjugation and Parity

- In the strong and EM interaction $C$ and $P$ are conserved separately.
  - In the weak interaction we know that $C$ and $P$ are not conserved separately.
    - The combination of $CP$ should be conserved!

- Consider how a neutrino (and anti-neutrino) transforms under $C$, $P$, and $CP$.
  - Experimentally we find that all neutrinos are left handed and anti-neutrinos are right handed.

$CP$ should be a good symmetry.
Neutral Kaons and $CP$ violation

- In 1964 it was discovered that the decay of neutral kaons sometimes ($10^{-3}$) violated $CP$!
  - The weak interaction does not always conserve $CP$!
  - In 2001 $CP$ violation was observed in the decay of $B$-mesons.
- $CP$ violation is one of the most interesting topics in physics:
  - The laws of physics are **different** for particles and anti-particles!
  - What causes $CP$ violation?
    - It is included into the Standard Model by Kobayashi and Maskawa (Nobel Prize 2008).
    - Is the $CP$ violation observed with $B$’s and $K$’s the same as the cosmological $CP$ violation?
- To understand how $CP$ violation is observed with $K$’s and $B$’s need to discuss **mixing**.
  - Mixing is a QM process where a particle can turn into its anti-particle!
  - As an example, lets examine neutral kaon mixing first ($B$-meson mixing later):
    - $K^0 = \bar{s}d, \bar{K}^0 = s\bar{d}$
    - In terms of quark content these are particle and anti-particle.
    - The $K^0$ has the following additive quantum numbers:
      - strangeness = +1
      - charge = baryon # = lepton # = charm = bottom = top = 0
      - $I_3 = -1/2$
    - The $K^0$’s isospin partner is the $K^+$ ($I_3$ changes sign for anti-particles.)
    - The $K^0$ and $\bar{K}^0$ are produced by the strong interaction and have definite strangeness.
    - They cannot decay via the strong or electromagnetic interaction.
Neutral Kaons, Mixing, and $CP$ violation

- The neutral kaon decays via the weak interaction, which does not conserve strangeness.
  
  - Let’s assume that the weak interaction conserves $CP$.
    
    - The $K^0$ and $\bar{K}^0$ are not the particles that decay weakly since they are not $CP$ eigenstates:
      
      $\begin{align*}
      P\left|K^0\right\rangle &= -\left|K^0\right\rangle \\
      P\left|\bar{K}^0\right\rangle &= -\left|\bar{K}^0\right\rangle \\
      C\left|K^0\right\rangle &= -\left|K^0\right\rangle \\
      C\left|\bar{K}^0\right\rangle &= -\left|\bar{K}^0\right\rangle \\
      CP\left|K^0\right\rangle &= \left|\bar{K}^0\right\rangle \\
      CP\left|\bar{K}^0\right\rangle &= \left|K^0\right\rangle
      \end{align*}$
    
    - We can make $CP$ eigenstates out of a linear combination of $K^0$ and $\bar{K}^0$:
      
      $\begin{align*}
      \left|K_1\right\rangle &= \frac{1}{\sqrt{2}}\left(\left|K^0\right\rangle + \left|\bar{K}^0\right\rangle\right) \\
      \left|K_2\right\rangle &= \frac{1}{\sqrt{2}}\left(\left|K^0\right\rangle - \left|\bar{K}^0\right\rangle\right)
      \end{align*}$

    - If $CP$ is conserved in the decay of $K_1$ and $K_2$ then we expect the following decay modes:
      
      - $K_1 \rightarrow$ two pions ($\pi^+\pi^-$ or $\pi^0\pi^0$) ($CP = +1$ states)
      - $K_2 \rightarrow$ three pions ($\pi^+\pi^0\pi^0$ or $\pi^0\pi^0\pi^0$) ($CP = -1$ states)

      ★ In 1964 it was found that every once in a while ($\approx 1/500$) $K_2 \rightarrow$ two pions!
Neutral Kaons

- The $K^0$ and $\bar{K}^0$ are eigenstates of the strong interaction.
  - These states have definite strangeness, are not $CP$ eigenstates
  - They are particle/anti-particle.
  - They are produced in strong interactions (collisions) e.g.
    \[ \pi^- p \rightarrow K^0 \Lambda \text{ or } K^0 \Sigma^0 \]

- The $K_1$ and $K_2$ are eigenstates of the weak interaction, assuming $CP$ is conserved.
  - These states have definite $CP$ but are not strangeness eigenstates.
  - Each is its own anti-particle.
  - These states decay via the weak interaction and have different masses and lifetimes.
    \[ K_1 \rightarrow \pi^0 \pi^0 \quad K_2 \rightarrow \pi^0 \pi^0 \pi^0 \]

- 1955: Gell-Mann/Pais pointed out there was more decay energy (phase space) available for $K_1$ than $K_2$.
  - $K_1$ and $K_2$ lifetimes should be very different.
    \[ m_K - 2m_\pi = 219 \text{ MeV/c}^2 \quad m_K - 3m_\pi \approx 80 \text{ MeV/c}^2 \]
  - Expect $K_1$ to have the shorter lifetime.
  - The lifetimes were measured to be:
    \[ \tau_1 \approx 9\times10^{-11} \text{ sec} \quad (1947-53) \]
    \[ \tau_2 \approx 5\times10^{-8} \text{ sec} \quad (\text{Lande et. al., Phys. Rev. V103, 1901 (1956)}) \]

- We can use the lifetime difference to produce a beam of $K_2$’s from a beam of $K^0$’s.
  - Produce $K^0$’s using $\pi p \rightarrow K^0 \Lambda$.
  - Let the beam of $K^0$’s propagate in vacuum until the $K_1$ component dies out.
    - 1 GeV/c $K_1$ travels on average $\approx 5.4 \text{ cm}$
    - 1 GeV/c $K_2$ travels on average $\approx 3100 \text{ cm}$
  - Need to put detector 100-200 m away from target.
Mixing and $CP$ violation

- How do we look for $CP$ violation with a $K_2$ beam?
  - Look for decays that have $CP = +1$:
    \[ K_2 \to \pi^+\pi^- \text{ or } \pi^0\pi^0 \]
  - Experimentally we find:
    - $K_2 \to \pi^+\pi^- \sim 0.2\%$ of the time
    - $K_2 \to \pi^0\pi^0 \sim 0.1\%$ of the time
  - Can also look for differences in decays that involve matter and anti-matter:
    \[ \delta(e) = \frac{BR(K_2 \to \pi^-e^+\nu_e) - BR(K_2 \to \pi^+e^-\overline{\nu_e})}{BR(K_2 \to \pi^-e^+\nu_e) + BR(K_2 \to \pi^+e^-\overline{\nu_e})} = 0.333 \pm 0.014 \]
    \[ \delta(\mu) = \frac{BR(K_2 \to \pi^-\mu^+\nu_\mu) - BR(K_2 \to \pi^+\mu^-\overline{\nu}_\mu)}{BR(K_2 \to \pi^-\mu^+\nu_\mu) + BR(K_2 \to \pi^+\mu^-\overline{\nu}_\mu)} = 0.303 \pm 0.025 \]
    - Nature differentiates between matter and antimatter!

- $CP$ violation has recently (2001) been unambiguously measured in the decay of $B$-mesons.
  - This is one of the most interesting areas in HEP (will be discussed later).
  - Observing $CP$ violation with $B$-mesons is much more difficult than with kaons!
    - $B_S$ and $B_L$ have essentially the same lifetime
      - No way to get a beam of $B_L$ and look for “forbidden” decay modes.
    - It is much harder to produce large quantities of $B$-mesons than kaons.

Christenson et. al. PRL V13, 138 (1964)