Physics 131

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Office hours: 1:30-2:30pm on Tues and Thurs

• Today handouts:
  ■ General Information for Physics 131-3
  ■ Assignment Sheet
    ✦ specific info on 131
    ✦ schedule for course, homework, midterms, final
  ■ Equation Sheet
  ■ Lecture notes (Chapters 1 and 2)
    ✦ distributed as a courtesy for those who attended the lectures

■ Goal:
  ✦ learn basic mechanics with everyday applications for scientists and engineers
good understanding of basic mechanics
give you competitive advantage over your colleagues in the work place

1. Measurement

Units:
• all measurements must have a unit
• units give physical meanings to numbers

e.g. I drove to take the P131 final at 70
   -- unclear what you mean by “70”
   I drove to take the P131 final at 70 miles per hour
   -- miles per hour (mph) is the unit

System of Units:
We will use the SI units:

<table>
<thead>
<tr>
<th>Base</th>
<th>SI</th>
<th>English</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time</td>
<td>second (s)</td>
<td>second (s)</td>
</tr>
<tr>
<td>Length</td>
<td>meter (m)</td>
<td>feet (ft)</td>
</tr>
<tr>
<td>Mass</td>
<td>kilogram (kg)</td>
<td>slug</td>
</tr>
</tbody>
</table>
• use these 3 base units to define other useful units:
  e.g. speed + length/ time
     : meter/ second (m/ s)

• for convenience, use units in powers of 10 of base units:
  e.g. 1000 meter (m) = 1 kilometer (km)
       (kilo = 1000)
  1 kilogram (kg) = 1000 grams (g)
  1 centimeter (cm) = \( \frac{1}{100} \) meter (centi = \( \frac{1}{100} \))

**Unit Conversion:**
Sometimes we need to convert from one unit to other:
  e.g. converse 60 miles into SI unit:
       60 miles \( \times \frac{1.61 \text{ km}}{1 \text{ mile}} \times \frac{1000 \text{ m}}{1 \text{ km}} = 96,600 \text{ m}\)

**Scientific Notation:**
Use for very small or large numbers:
2. Straight Line Motion

• study motion in a straight line
  i.e. one dimensional motion

• describe the motion using displacement, velocity, acceleration

Consider walking across the lecture hall:
The computer produces the following sketch of $x$ vs $t$:

**Displacement:**
the change from one position to another:
$\Delta x = x_2 - x_1$

For the example on walking:
\[ \Delta x = x_2 - x_1 \]
\[ = 10 - 0 \]
\[ = 10 \text{ m} \]

Question: If I return to the detector after pause for 60 s, what is the displacement between the beginning and end of the walk?

\[ \Delta x = x_2 - x_1 \]
\[ = 0 - 0 \]
\[ = 0 \text{ m} \]

\( \star \) displacement is not same as distance traveled. In the example, distance traveled is 20 m but displacement is zero.

• To describe how fast I walked, we define

\( \star \) average velocity \( \bar{v} = \frac{\text{displacement}}{\text{time}} \)

\[ \bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1} \]

\( \star \) average speed \( \bar{s} = \frac{\text{distance traveled}}{\text{time}} \)

\[ \bar{s} \geq 0 \]
In the example on walking:

- for the 1st segment (detector furthest point)
  \[ v = \frac{10 - 0}{10 - 0} = 1 \text{ m/s} \]

- for the round trip (detector furthest point detector)
  \[ v = \frac{0}{10 + 60 + 15} = 0 \text{ m/s} \]
  -- time spent at furthest point is included in \( \Delta t \)

- for the round trip
  \[ s = \frac{10 + 10}{10 + 60 + 15} = 0.24 \text{ m/s} \]

☆ Zero average velocity does not necessary implies a zero average speed.
☆ We usually refer to the average speed in everyday conversation.
Instantaneous Velocity and Speed:

Consider the $x$ vs $t$ plot recorded by the computer for the walk:

![Graph](image)

- The average velocity is just the slope of the line connecting two positions.
  - e.g. the line connecting $x = 10$ m and 0 m
  
  $$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{0 - 10}{85 - 70} = -\frac{10}{15} = -0.66 \text{ m/s}$$
  
  -- this average velocity include the 5 s stoppage!

If we calculate the average velocity over a very small time interval

- instantaneous velocity $\nu = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$
\[ \frac{dx}{dt} \]

- A directional quantity
  - i.e. \( v > 0 \) corresponds to moving in the \( +x \) direction and vice versa
- When we talk about “velocity”, we mean instantaneous velocity
- Instantaneous speed = magnitude of instantaneous velocity
  \[ s = |v| \]
  - No associated direction
- Car speedometer measures the speed

**Velocity vs Time Plot:**
Consider a car traveling on a highway at 100 km/h. Because of construction, the car must come to a complete stop, then move with a velocity of 20 km/h before resuming the 100 km/h speed. The position vs time plot looks like:
Since the velocity at a given time is just the slope of the tangent to the curve, we can measure the slope and produce the following velocity vs time plot:

Two special features:
- the horizontal lines (zero slope) correspond to constant velocity
• the vertical lines (infinite slope) corresponds to instantaneous changes of velocity, e.g. from certain velocity to zero in no time
  • infinite “acceleration” and “deceleration”

**Acceleration:**
-- rate of change of velocity

Average acceleration:
\[
\overline{a} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}
\]

Instantaneous acceleration:
\[
a = \lim_{{\Delta t \to 0}} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \frac{d^2 x}{dt^2}
\]

• a directional quantity
  i.e. \( a > 0 \) corresponds to accelerating in the \( +x \) direction and vice versa.
• \( a > 0 \) can mean “acceleration” or
“deceleration”, i.e. can’t tell “acceleration” or “deceleration” from the sign of $a$.

- If $v$ and $a$ have same sign
  - $v$ and $a$ point in same direction
  - acceleration

- If $v$ and $a$ have opposite sign
  - $v$ and $a$ point in opposite direction
  - deceleration

A more realistic $v$ vs $t$ plot:
Velocity

accelerating to full speed

slowing down (deaccelerating)

speeding up from stop

From the slope of the line $a$ vs $t$ plot:

velocity is constant, $a = 0$
### Review:

<table>
<thead>
<tr>
<th>Definition</th>
<th>Units</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>displacement $\Delta x = x_2 - x_1$</td>
<td>$L$</td>
<td>directional</td>
</tr>
<tr>
<td>distance trvl. path length $\frac{\Delta x}{\Delta t}$</td>
<td>$L$</td>
<td>$&gt;0$</td>
</tr>
<tr>
<td>avg. velocity</td>
<td></td>
<td>directional</td>
</tr>
<tr>
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### Constant Acceleration:

- Plan: find the equation of motion for a particle under constant acceleration, $a(t) = a(0) = \text{constant}$
  - i.e. a horizontal line in $a$ vs. $t$ plot

### First Equation of Motion:

Start with the definition:

\[
\begin{align*}
a & = \frac{v - v_0}{t - 0} \\
at & = v - v_0 \\
v & = v_0 + at
\end{align*}
\]

(1)
for $a = 0, \nu = \nu_0 = \text{constant}$

Second Equation of Motion:

Start with the definition:

$$\nu = \frac{x - x_0}{t - 0}$$

$$\nu t = x - x_0$$

$$x = x_0 + \nu t$$

For constant acceleration, we have a straight line in the $\nu$ vs. $t$ plot:

\[ \nu \text{ is just halfway between } \nu_0 \text{ and } \nu: \]

$$\bar{\nu} = \frac{\nu + \nu_0}{2}$$

$$\Rightarrow x = x_0 + \frac{1}{2}(\nu + \nu_0)t$$

(2)
Third Equation of Motion:
Substituting (1) into (2):
\[ x = x_0 + \frac{1}{2} (v_0 + at + v_0) t \]
\[ \Rightarrow x = x_0 + v_0 t + \frac{1}{2} at^2 \quad (3) \]
Use this to calculate the position at time \( t \) if you know the acceleration \( (a) \), initial position \( (x_0) \), and initial velocity \( (v_0) \).

Fourth Equation of Motion:
From (1):
\[ v = v_0 + at \]
\[ \Rightarrow t = \frac{v - v_0}{a} \]
Substituting into (3):
\[
x = x_0 + v_0 \frac{v - v_0}{a} + \frac{1}{2} a \left( \frac{v - v_0}{a} \right)^2
\]

\[
= x_0 + \frac{vv_0}{a} - \frac{v_0^2}{a} + \frac{v^2}{2a} + \frac{v_0^2}{2a} - \frac{vv_0}{a}
\]

\[
= x_0 - \frac{v_0^2}{2a} + \frac{v^2}{2a}
\]

\[
\Rightarrow v^2 = v_0^2 + 2a(x - x_0)
\]

Use this to calculate the velocity at position \(x\) if you know the acceleration \((a)\), initial position \((x_0)\), and initial velocity \((v_0)\).

Summary:

<table>
<thead>
<tr>
<th>Equation</th>
<th>Missing Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>(v = v_0 + at)</td>
<td>(x, x_0)</td>
</tr>
<tr>
<td>(x = x_0 + v_0t + \frac{1}{2} at^2)</td>
<td>(v)</td>
</tr>
<tr>
<td>(v^2 = v_0^2 + 2a(x - x_0))</td>
<td>(t)</td>
</tr>
<tr>
<td>(x = x_0 + \frac{1}{2} (v_0 + v)t)</td>
<td>(a)</td>
</tr>
<tr>
<td>(x = x_0 + vt - \frac{1}{2} at^2)</td>
<td>(v_0)</td>
</tr>
</tbody>
</table>
Problem 44E (p. 31):

A P131 student is driving at 85 mi/h on a highway, and spots a state trooper. If his brakes are capable of decelerating at 17 ft/s², what is the minimum time to bring the car under the 55 mi/h limit?

Convert all quantities into the same unit:

\[ a = -17 \text{ ft/s}^2 \times \frac{1 \text{ mi}}{5280 \text{ ft}} = -0.0032 \text{ mi/s}^2 \]

\[ v_0 = 85 \text{ mi/h} \times \frac{1 \text{ h}}{3600 \text{ s}} = 0.024 \text{ mi/s} \]

\[ v = 55 \text{ mi/h} \times \frac{1 \text{ h}}{3600 \text{ s}} = 0.015 \text{ mi/s} \]

\[ t = \frac{v - v_0}{a} \]

\[ = \frac{0.015 \text{ mi/s} - 0.024 \text{ mi/s}}{-0.0032 \text{ mi/s}^2} \]

\[ = 2.8 \text{ s} \]

✓ The answer has the proper units.
✓ The answer is reasonable.
Problem 56P (p. 32):

When a traffic light turns green, a car starts with constant acceleration $a = 2.2 \text{ m/ s}^2$. At the same time, a truck traveling with a constant speed of 9.5 m/ s overtakes the car. (a) Where will the car overtake the truck? (b) What is the speed then?

Draw a diagram and write down all known quantities:

- **Truck**: $v_{\text{truck}} = 9.5 \text{ m/ s}$
  - $x = x_0 + v_{\text{truck}} t = v_{\text{truck}} t$

- **Car**: $v_0 = 0$, $a = 2.2 \text{ m/ s}^2$
  - $x = x_0 + v_0 t + \frac{1}{2} at^2 = \frac{1}{2} at^2$
  - $v_{\text{truck}} t = \frac{1}{2} at^2$
  - $2v_{\text{truck}} = t$
  - \[ t = \frac{2 \times 9.5 \text{ m/ s}}{2.2 \text{ m/ s}^2} \]
  - \[ = 8.6 \text{ s} \]
Substituting in (1):

\[ x = v_{\text{truck}} t = (9.5 \text{ m/s})(8.6 \text{ s}) = 82 \text{ m} \]

\[ v = v_0 + at = 0 + (2.2 \text{ m/s}^2)(8.6 \text{ s}) = 19 \text{ m/s} \]

✓ The answer has the proper units.
✓ \( v > v_{\text{truck}} \) when overtaken.

**Free-Fall Acceleration:**

We will study the motion of freely falling objects near the Earth's surface (several miles), neglecting air resistance.

The new equations of motion, with \( a = -g \):

\[ y = y_0 + v_0 t - \frac{1}{2} g t^2 \]

\[ v = v_0 - g t \]

\[ v^2 = v_0^2 - 2g(y - y_0) \]
Example:
A rock falls off a cliff of height $h$. (a) How long does it take to hit the ground? (b) What is the velocity of the rock at that instant?
\[ y_0 = h, \; v_0 = 0 \]
\[ y = 0, \; t = ?, \; v = ? \]
\[ y = y_0 + v_0 t - \frac{1}{2} gt^2 \]

\[ \Rightarrow 0 = h - \frac{1}{2} gt^2 \]

\[ \Rightarrow t = \sqrt{\frac{2h}{g}} \]

\[ v = v_0 - gt \]

\[ \Rightarrow v = -g \sqrt{\frac{2h}{g}} \]

\[ \Rightarrow v = -\sqrt{2gh} \]

✓ Unit: \[ t = \sqrt{\frac{\text{length}}{\text{length} / \text{time}^2}} = \text{time} \]

✓ Unit: \[ v = \sqrt{\frac{\text{length}}{\text{time}^2}} \times \text{length} = \frac{\text{length}}{\text{time}} \]

✓ \[ t > 0 \]

✓ \[ v < 0 \]

✓ \[ t = 0 \; \text{and} \; v = 0 \; \text{for} \; h = 0 \]
• The problem is solved using algebra, i.e. without numbers. This is the preferred technique.
• The solution is for a general problem, i.e. valid for any falling object at rest from a height $h$.
• The solution is independent of the mass of the object!

Question:
- If I drop a ball and a piece of paper from the same height, which object will hit the ground first?
- If I repeat the experiment with the paper balled up, which object will hit the ground first?

Conceptual Question:
If you drop an object in the absence of air resistance, it accelerates downward at 9.8 m/ s$^2$. If instead you throw it downward, its downward acceleration after the release is:

(1) $< 9.8$ m/ s$^2$
Comment:
If we toss an object upward, it will reach a maximum height and will fall down. At any given height, the speed of the object is the same, whether it is travelling upward or downward.