Problem Set 5
Due May 19, 2009

1) Taylor, Problem 8.24, page 205.

4) A set of \( n \) data points \( (x_i, y_i \pm \sigma_i) \) are related by: \( y = A + 5x \).
   a) Use the method of Least Squares to show that the best estimate of the intercept, \( A \), is given by:
   \[
   A = \frac{\sum_{i=1}^{n} y_i / \sigma_i^2 - 5 \sum_{i=1}^{n} x_i / \sigma_i^2}{\sum_{i=1}^{n} 1 / \sigma_i^2}
   \]
   b) Use propagation of errors to show that the variance of \( A \) is given by:
   \[
   \sigma_A^2 = \frac{1}{\sum_{i=1}^{n} 1 / \sigma_i^2}
   \]

5) In February 1987 a supernova was discovered by astronomers using optical telescopes. The supernova was close enough to earth that it was hoped that the neutrinos emitted in the explosion could be detected on earth. The following table gives the number of neutrino events detected in 10 second intervals by the IMB experiment (located in a salt mine near Cleveland) on 23 February 1987 -- around which time the supernova S1987a was first seen by experimenters:

<table>
<thead>
<tr>
<th># events</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td># intervals</td>
<td>1042</td>
<td>860</td>
<td>307</td>
<td>78</td>
<td>15</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Prediction</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(a) Ignoring the one 10 second interval with nine events, compute the average number events expected in an interval and fill in the predictions on the number of intervals to observe the number of events listed in the table.
(b) Justify that the interval with nine events is consistent with supernova explosion by computing the \( \chi^2 \) per degree of freedom and the corresponding probability with and without the interval with nine events.

NOTE: Assume the data can be described by a Poisson distribution. Also, the example on P276 of Taylor may be helpful for this problem.