1) The probability density function (pdf) for an electron in the lowest energy level (n = 1) state of a hydrogen atom, as a function of radial distance (r) from the nucleus, is given by:

\[ p(r) = \frac{4}{a^3} r^2 e^{-2r/a} \]  

with \( a \) = constant (know which one?)

a) Show that this is a properly normalized pdf.
b) What is the most probable radial distance (in terms of \( a \)) of the electron?
c) What is the average radial distance (in terms of \( a \)) of the electron?

2) We wish to determine the acceleration due to gravity (\( g \)) using the following data and \( h = 0.5gt^2 \).

a) Use the least squares technique to find the best value of \( g \). Assume the error in each \( h \) (height) measurement is 0.01 m and the time is measured exactly. (See Taylor Problem 8.5)

<table>
<thead>
<tr>
<th>h (m)</th>
<th>t (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>0.1</td>
</tr>
<tr>
<td>0.44</td>
<td>0.3</td>
</tr>
<tr>
<td>1.23</td>
<td>0.5</td>
</tr>
<tr>
<td>2.40</td>
<td>0.7</td>
</tr>
</tbody>
</table>

b) What is the value of the chi-square (\( \chi^2 \)) for this problem?
c) How many degrees of freedom are there in this problem? (See Taylor Problem 12.14, part b))
d) Estimate the probability to get a \( \chi^2 \) per degree of freedom \( \geq \) what you obtain using parts b) and c).


4) Two different experiments have measured the mass of the Ohio boson. Experiment #1 measured 1.00 ± 0.01 gm while experiment 2 measured 1.04 ± 0.02 gm.

a) What is the best estimate of the mass of the Ohio boson if we combine the two experiments?
b) Calculate the \( \chi^2 \) for the two measurements in this problem using:

\[ \chi^2 = \sum \frac{(m_i - m)^2}{\sigma_i^2} \]

with \( m_i \) the measurement from experiment \( i \) and \( \sigma_i \) the standard deviation of the measurement, and \( m \) the best estimate of the mass obtained by combining the two experiments.
c) How many degrees of freedom are there for this \( \chi^2 \)?
d) What's the probability of getting a value of \( \chi^2 \) per degree of freedom \( \geq \) to the one in this problem?


6) Taylor, Problem 12.8, page 280.

7) A theory states that the angular distribution of electrons from the decay of an unstable particle should have a probability distribution function of the form (both \( N \) and \( \alpha \) are constants):

\[ p(\cos \theta) = N(1 + \alpha \cos^2 \theta) \]

An experiment measures ten examples of the decay of this unstable particle and finds the following values of \( \cos \theta \): (-0.05, -0.15, -0.25, -0.35, 0.45, 0.55, 0.65, 0.75, 0.85, 0.95). For this problem the limits on \( \cos \theta \) are [-1, 1]. We wish to determine the value of \( \alpha \) using the Maximum Likelihood Method.

a) Use the normalization condition for a probability distribution function to show that:

\[ N = \frac{1}{2(1 + \alpha / 3)} \]

b) Write down the Likelihood Function for this problem.
c) Make a plot of the Likelihood Function vs. \( \alpha \) for -1.5 < \( \alpha \) < 1.5. Use this plot to find the value of \( \alpha \) that maximizes the Likelihood Function.